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Quantification of Selection Criteria for Reliability Improvement Warranty Contracts.

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QUANTIFICATION OF SELECTION CRITERIA FOR RELIABILITY IMPROVEMENT WARRANTY CONTRACTS

A DISSERTATION

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ABSTRACT

Two models are developed in the study to extend the methodology presently available for the selection of equipment to be purchased with a Reliability Improvement Warranty (RIW).

The first model determines the economic value of a RIW contract. Finding the economic value of a RIW contract involves computing the difference between the Life Cycle Cost for the non-RIW alternative and the RIW alternative.

The second model utilizes the theory of discriminant analysis to judge whether to employ a RIW in a procurement. The data base for the necessary population of items is generated by employing the Monte Carlo method. Items are separated into the non-RIW or RIW alternative subpopulations by using the economic value of a RIW contract model.

Conclusions drawn from this study are as follows:

1. Three possible approaches result for deciding whether to employ a RIW contract.
    a. Approach one requires the use of the economic value model.
    b. Approach two requires the use of a linear discriminant function to classify items as applicable or not applicable for a RIW.
    c. Approach three requires the determination of an interval cut-point by applying the linear discriminant function found in approach two. Unclassified items with

discriminant functional values falling outside of the interval are classified into their appropriate groups. Unclassified items with discriminant functional values inside the interval are classified by employing the economic value of a RIW model.

2. The three approaches developed to classify items are independent of the avionic industry.

3. The economic value of a RIW model provides estimates of various costs and reliability parameters associated with the warranty.

4. The economic value model can be employed to obtain the optimal or most cost-effective warranty period.

5. The warranty period should be long enough to permit the product to reach a state of design stability so that specified reliability improvements can be employed.

6. The economic value of a RIW model simplifies the task of warranty pricing.

7. The cut-point developed in the discriminant analysis model represents a break-even point for the application of RIW contracts.

CHAPTER 1

INTRODUCTION

In 1968 the United States Navy entered into the first military Reliability Improvement Warranty contract. The United States Air Force first applied the concept in 1969. Subsequently various branches of the Department of Defense have utilized the Reliability Improvement Warranty Contract on a trial basis in approximately ten instances. Although the results of some of these contracts are still pending it is generally felt that the Reliability Improvement Warranty has been effective in increasing reliability and reducing repair costs.[1]

This work deals with an extension of the methodologies presently available to the buyer with respect to application of the Reliability Improvement Warranty concept in the equipment procurement process.[2]

RELIABILITY IMPROVEMENT WARRANTY

An understanding of the concept of the Reliability Improvement Warranty (RIW) is dependent upon the concepts of warranties and Life

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[1]Klass, Philip J., "Failure Free Warranty Idea Lauded, Wider Use Deserved," Aviation Week and Space Technology, Feb. 9, 1970, p. 57.

[2]The topic was obtained from the Air Force Research Management Center at Wright-Patterson Air Force Base, Ohio.

Cycle Costing. Fundamentally a Reliability Improvement Warranty is a warranty which employs the Life Cycle Costing procurement principle.

Warrantles

A warranty is a device used to protect a buyer from unidentified defects in the supplies or services provided by a seller. At the same time the warranty limits the liability of the seller.[3]

In years past, courts seemed to recognize only expressed warranties--those stated in written or spoken words. Usually these were quite limited in coverage and seemed intended to protect the seller from the buyer's claim. Increasing complaints have led to the development of legislation to protect the consumer in many areas, one of which is product reliability. As a result the definition of warranty coverage has been broadened to encompass the concept of an "implied warranty." The idea behind this type of warranty is that a warranty is implicitly intended by the seller even in those cases where it is not explicitly stated. The Uniform Commercial Code[4] states that:

> Where the seller at the time of contracting has reason to know any particular purpose for which the goods are required and that the buyer is relying on the seller's skill or judgment to select or furnish

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[3]Allen, Dennis Jean, "Application of the Reliability Warranty to Department of Defense Procurements" (unpublished Master's thesis, Navy Postgraduate School, 1975), p. 10.

[4]The Uniform Commercial Code is used as the basis for transacting public and private contracts. Because federal law is consistent with the Uniform Commercial Code, the Government has followed this code in preparing contracts.

> suitable goods, there is, unless excluded or modified under the next section, an implied warranty that the goods shall be fit for such purpose.[5]

The Air Force has utilized warranties on equipment purchases since it became a separate service. Traditionally the primary purpose of a warranty has been to assure correction of latent defects in purchased items. In recent years warranty clauses are being used to cover longer periods of time with the goal of achieving a desired performance level. Problems associated with the enforcement of such warranties has led to the development of the Reliability Improvement Warranty concept.[6]

In 1966 Lear Seigler Inc. proposed to the Navy the first RIW contract.[7] The first use of this concept in an Air Force contract occurred in 1969 between Lear Seigler Inc. and the Air Force Systems Command for 128 F-111 A24G-26 displacement gyros.

RIW differs from the general warranty because it goes beyond the normal protection against defective or non-conforming equipment. It is a contractual arrangement which is employed as a method of assuring a given level of reliability. The United States Air Force

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[5]American Law Institute and National Conference of Uniform State Laws, Uniform Commercial Code, West Publishing Co., 1972 Official Text, p. 86.

[6]Nixon, Harvey L., Jr. and Christopher B. Hitchcock, "A Simulation of the Reparable Processing Procedures Applicable to Reliability Improvement Warranties" (unpublished Master's thesis, School of Systems and Logistics, Air Force Institute of Technology, 1973), p. 7.

[7]The first RIW contracts were called "failure free" warranties. The term "failure free" warranty proved to be misleading in two ways. It implied the warranty was free from failure and that the warranty was free from cost. Recognizing that it was confusing, it was replaced with the term Reliability Improvement Warranty.

defines RIW as:

> ...a provision in either a fixed price acquisition, or a fixed price equipment overhaul contract in which:
>
> a. the contractor is provided with a monetary incentive, throughout the period of the warranty, to improve the production design and engineering of the equipment so as to enhance the field/operational reliability and maintainability of the system/equipment; and
>
> b. the contractor agrees that, during a specified or measured period of use, he will repair or replace (within a specified turnaround time) all equipment that fails (subject to specified exclusions, if applicable).[8]

It is important to note initially that a Reliability Improvement Warranty is not a warranty in the classic sense with respect to materials and workmanship. RIW differs from the traditional warranty in several respects. It calls for the manufacturer to replace or repair, at his option, any warranted unit within a specified time unit (in operating hours, calendar time or both), except in cases of obvious misuse. The contract establishes a fixed price for a given level of performance. This price is based upon the anticipated number of failures and the cost of each repair action. The anticipated number of failures over the warranty period are determined by assuming that the reliability of the warranted

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[8] U.S. Department of the Air Force, Interim Guidelines: Reliability Improvement Warranty, Washington, D.C., 1974, pp. 5-6.

[9] U.S. Department of the Air Force, Reliability Improvement Warranties Description and Use, Business Research Management Center, Wright-Patterson AFB, Ohio, 1974, p. 2.

item will improve from an initial level to some specified level as a result of the contractor's planned reliability improvement program. The philosophy behind RIW is that once the fixed price warranty contract is established the profit realized by the contractor is dependent upon the equipment's reliability. Thus contractors are motivated to focus their attention on the reliability of the items under contract through the use of "no cost" (to the Government) engineering change proposals.[9]

As previously mentioned RIW coverage is described in either operating hours, calendar time or both. The use of calendar time protects the contractor's obligation in those cases where reliability forces the contractor to repair more units than expected. Conversely the use of operating hours benefits the Air Force in cases where the equipment is not operated as often as expected. Most of the contracts issued thus far have used both units of time.[10]

The nature of RIW requires that administrative procedures be efficient. These procedures include the collection of operational and failure times on each individual unit under contract. Information

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[9]U.S. Department of the Air Force, Reliability Improvement Warranties Description and Use, Business Research Management Center, Wright-Patterson AFB, Ohio, 1974, p. 2.

[10]U.S. Department of the Air Force, Interim Guidelines: Reliability Improvement Warranty, Washington, D.C, 1974, p. 16.

of this type is required so that the buyer and seller may communicate the terms of the contract.

Life Cycle Costing

Since early 1961 a continuous effort has been maintained by various manufacturers of military equipment to show to the Department of Defense that the "low initial price" procurement criteria was more often than not the most expensive way to buy. As an alternative to the "low initial price" procurement these manufacturers developed the concept of Life Cycle Costing. Life Cycle Costing is defined as a procurement technique having as its objective the making of competitive awards of military contracts on the basis of lowest total cost of ownership to the Government. The total Life Cycle Cost or total cost of ownership is defined to include operating, maintenance, and other costs as well as the cost of acquisition.[11]

Studies have revealed certain relationships between Life Cycle Cost and reliability. Throughout this paper the measure of effectiveness for "reliability" will be expressed in terms of "Mean Time Between Failures (MTBF)." It is expected in general that Life Cycle Cost will vary inversely with reliability (MTBF). Figure 1 illustrates this relationship. For most complex equipment, as reliability increases the Life Cycle Cost decreases due to the reduction of maintenance cost. It should be noted that there exists

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[11]Markowitz, Oscar, "When Purchasing and Design Look to Total Costs," Purchasing Magazine, June, 1970, pp. 65-69.

a point where the Life Cycle Cost will begin to increase again as MTBF increases. That is, there is a point of negative returns on reliability improvement effort.[12]

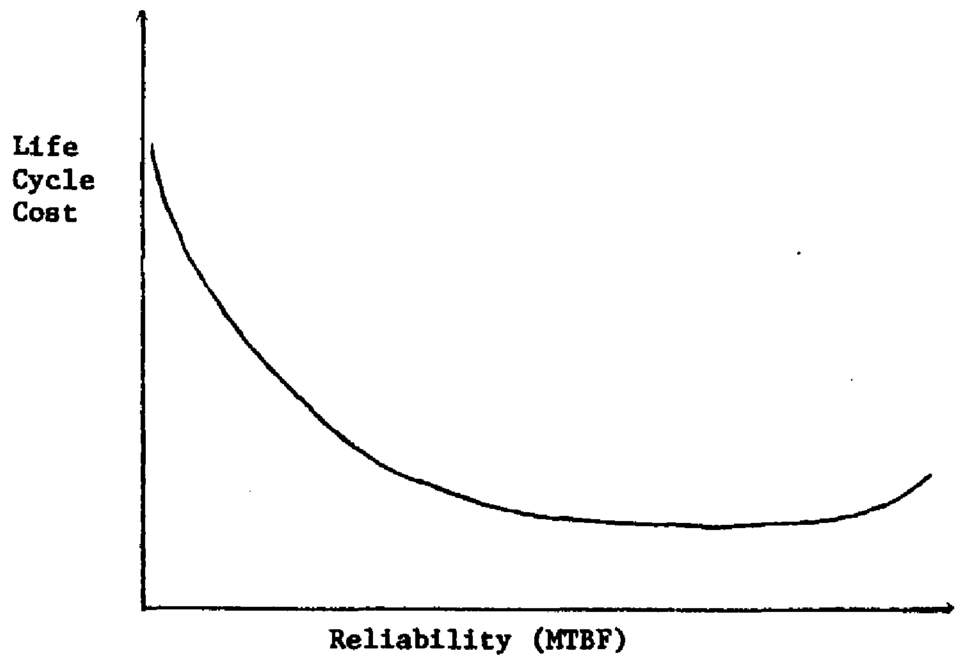

Figure 1

Life Cycle Cost Versus Reliability

For most Air Force equipment the procurement objective is to obtain more reliable products from vendors. Unfortunately the strategy of the seller opposes the strategy of the buyer. Balaban and Retterer concluded:

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[12] Dunn, Payton E. and Andrew A. Oltyan, "Evaluation of Proposed Criteria to Be Used in the Selection of Candidates for Reliability Improvement Warranties" (unpublished Master's thesis, School of System and Logistics, Air Force Institute of Technology, 1975), p. 5.

> Today's procurement practice, with the emphasis on low initial purchase price, causes vendors to supply the lowest reliability that will pass the procurement acceptance requirements. The vendor is economically driven to this position, his maximum profit being derived from such strategy.[13]

Figure 2 shows the current profit-reliability curve and the desired profit-reliability curve.

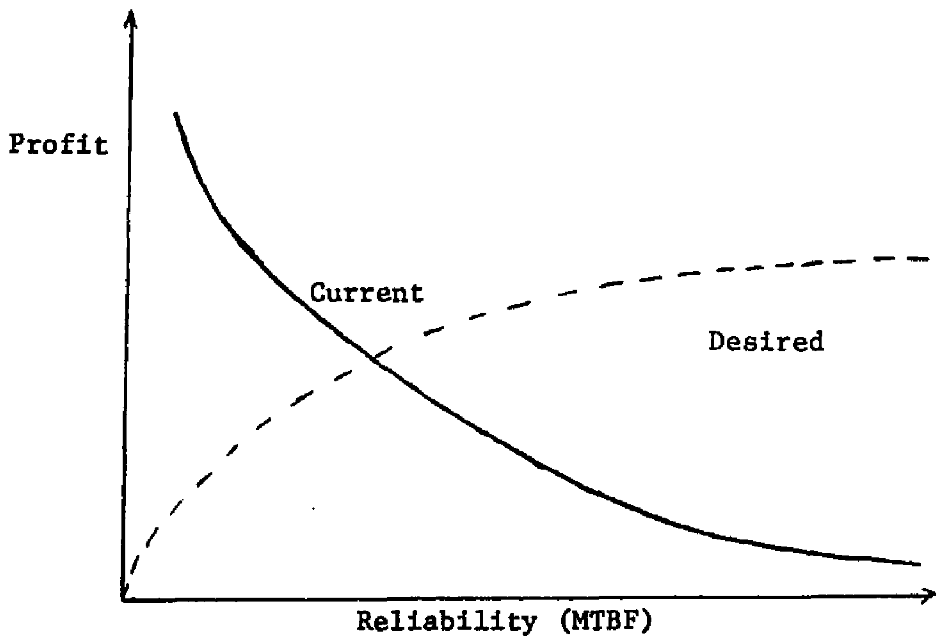

Figure 2

Profit Versus Reliability

To illustrate the fundamentals of an RIW contract let us consider the basics of the first contract between Lear Seigler Inc. and the U.S. Navy in 1967. The contract involved a fixed price to

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[13] Balaban, Harold S. and Bernard L. Retterer, "The Use of Warranties for Defense Avionics Procurement," The ARINC Research Corporation Report 0637-02-1-243, The ARINC Research Corporation, Annapolis, Maryland, June, 1973, p. 4.

cover all the overhauls required on a long term field operating period for the CN-494A/AJB-3 gyroscope platform. Historically this gyro platform was an item with a high failure rate. The manufacturer contended that he could improve the mean time between failures for the gyros if he were allowed to remain responsible for their overhaul over the long term. After considerable negotiation a contract was established as a 3,000 hour field operational warranty over a period of 5 years on 800 gyroscopes. The fixed price was based on a 30 percent improvement of reliability or a return of 2,400 failures to Lear Seigler Inc. over the contract period and an average overhaul cost of $1,000 per failure. As in all RIW contracts the contractor is motivated to increase reliability beyond that which was negotiated since every failure under 2,400 implies $1,000 less he will have to spend. In addition the risk of more than 2,400 failures occurring forces the contractor to monitor his reliability improvement program to meet the goals determined by the contract. The Department of Defense benefits because it has reduced its repair cost of the particular item over the contract period and ends up with a more reliable piece of equipment. In the case of the gyro platform the Government saved approximately $1,046,000 in Life Cycle Cost while increasing MTBF above the 30 percent specified improvement. Thus the RIW contract reduced the cost per operational hour of the gyro over the contract period.[14]

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[14] Markowitz, Oscar, "Life Cycle Costing Applied to the Procurement of Aircraft Spare Parts" (unpublished Master's thesis, Drexel University, Philadelphia, Pa., May, 1971), pp. 50-51.

A RIW provision may be introduced during initial design and production or even after procurement by the Government. The rule of introduction for RIW is the earlier the better during the procurement cycle. Obtaining contractor involvement during the initial stages is important because equipment generally undergoes a reliability growth process from the time of initial design until it reaches a state of maturity.[15] Insuring contractor involvement at the earliest possible stage theoretically accelerates reliability growth and minimizes the cost to achieve it. Thus when the purchaser takes over the risk and reliability of the equipment upon expiration of the warranty, they will hopefully be in an optimal position.

PROBLEM STATEMENT

The question of whether to apply the RIW in the procurement of an item is of utmost importance to the Department of Defense. On dispensable items it is of little value, but on other items it has tremendous value. This leads to the discussion of whether or not there is a break even point or under what conditions does it become useful to use a RIW contract. At present the methodology to use in making this decision is incomplete. Therefore it is proposed that a model be developed which will provide the procurement officer with a solution to this problem.

The current selection process requires that potential RIW candidates satisfy a set of application guidelines. Given below

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[15] Balaban and Retterer, op. cit., p. 16.

is a list of the guidelines.[16]

a. Units should be field-testable to avoid return of good units.

b. Units should be readily transportable.

c. Units should be self-contained and not dependent on auxiliary equipment.

d. Units should be sealed to avoid tampering.

e. Units should lend themselves to serial number management.

f. Moderate to high support costs should be involved.

g. Knowledge concerning the application of the unit in terms of expected operating time and the use environment should be available.

h. Units should have the potential for both reliability growth and the reduction in repair costs.

i. A sufficient quantity should be purchased to make RIW cost effective.

j. Units should have a reasonably high utilization level.

These guidelines serve as qualitative and quantitative conditions for application. Guidelines (a) through (e) are qualitative while guidelines (f) through (j) are quantitative in nature. Items do not have to satisfy all of the above guidelines in order to qualify for RIW application. The merits of an item relative to certain guidelines may be used to compensate for its weakness with respect to other guidelines.

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[16]Nixon and Hitchcock, op. cit., p. 4.

The procurement officer must first determine if the potential item for RIW application adequately satisfies the stated qualitative guidelines. Next he must determine on the basis of the quantitative guidelines whether or not a RIW procurement is in the long run economically attractive. Guidelines (f) through (j) are not adequate criteria to answer this question. It is here that the heart of the problem centers.

JUSTIFICATION

Inflation and a dwindling defense budget have prompted the Department of Defense to search for ways to reduce costs while maintaining a strong defense.[17] One approach is to attempt to reduce the cost of the maintenance of equipment. This type of expenditure is a function of the reliability of equipment.

Improving reliability serves as a means of reducing the cost of maintenance. In those cases applicable the RIW contract represents an investment which has as its goal the reduction of maintenance cost. A study by the Logistics Management Institute[18] on four aircraft system case studies demonstrated that an additional investment of 7.8% on reliability improvement programs could reflect a net savings

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[17] U.S. Department of the Navy, <u>Proceedings at Failure Free Warranty Seminar</u>, Dec. 12-13, 1973 (Philadelphia: U.S. Navy Aviation Supply Office, 1973), p. N-17.

[18] Logistics Management Institute, "Criteria for Evaluating Weapon System Reliability, Availability and Costs," LMI Task No. 73-11, Logistics Management Institute, Washington, D.C., March 1974, pp. 70-71.

of 27% in total cost. It is this type of return on investments that the Department of Defense needs in order to resolve the effects of inflation and a dwindling defense budget. Thus it is important that the methodology of when to apply a RIW be developed.

RESEARCH OBJECTIVE AND METHODOLOGY

The objective of this study is to extend the framework now available to the United States Air Force in determining whether to apply a Reliability Improvement Warranty for a specific item. The approach used to extend this framework is dependent upon two different procedures for classifying a piece of military equipment as applicable or not applicable for a RIW contract.

Approach number one involves defining a model for computing the economic value of a RIW contract as the difference between the Life Cycle Cost for a non-RIW procurement and the Life Cycle Cost for a RIW procurement. A non-negative economic value classifies an item as appropriate for a RIW contract while a negative economic value classifies the item as being not appropriate for such a RIW contract.

The second approach employs the multivariate statistical technique of discriminant analysis with respect to classifying items as RIW applicable or not. In order to apply this statistical classification technique a history of measures on key variables taken from a population of items which have been classified in either the RIW or non-RIW category must be known. Because the data base with respect to RIW applications is small due to the limited number of applications to date the following steps will be carried out to complete this model.

1. The Monte Carlo technique will be applied to generate a multivariate population of measures to represent a collection of hypothetical items for potential RIW application.
2. The economic value of a RIW model will be applied to the population generated in step 1 in order to classify each into the applicable or non-applicable for RIW subpopulations.
3. The discriminant analysis model is then applied to the two subpopulations of multivariate measures to obtain the necessary quantitative criteria for classifying equipment procurements as applicable or non-applicable for RIW contracts.

A limitation in both analytical approaches to the problem is that only approximately 10 RIW contracts have been executed within the Department of Defense. In view of this limited use of the RIW, any framework developed will have to await the future generation of data to be adequately tested. Despite this limitation the need for such a framework exists.

CHAPTER 2

ECONOMIC VALUE OF A RELIABILITY IMPROVEMENT WARRANTY

Even though RIW contracts offer a potential benefit to users, they are not applicable in every military procurement. Thus it is necessary that the procurement officer be able to determine whether a RIW is economically attractive for a particular item. In this study the economic value of a RIW contract is the saving (loss) of a RIW contract found by determining the difference between the Life Cycle Cost for the item procured without a RIW and the Life Cycle Cost for the item with a RIW, where both are measured over the given warranty period.

BASIC ASSUMPTIONS

In the development of the economic value model it is assumed that the item under consideration for a RIW contract has passed the qualitative guidelines for application mentioned in Chapter 1. Furthermore the economic value obtained from the model is to serve as a basis for determining if the quantitative guidelines given in Chapter 1 have been satisfied.

Two of the most noted issues in the mathematical theory of reliability are: (1) the choice of the distribution of failures; and (2) the assumption of independence of failures.[1]

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[1] Logistic Management Institute, "Criteria for Evaluating Weapon System Reliability, Availability and Costs," LMI Task No. 73-111, Logistics Management Institute, Washington, D.C., March 1974, p. 21.

One popular relationship between failure and equipment life is referred to as the bathtub curve. The bathtub curve given in the figure that follows plots the failure rate against the lifetime of the population of units purchased. The failure rate is initially high due to early failure, then it drops to become constant, and finally rises due to wearout failures. The flat portion of the bathtub curve, which represents the constant failure rate, makes up a major part of the curve. For our purpose it is reasonable to assume a constant failure rate initally. Early failure and wearout periods are ignored. This is logical in that the usual warranty (implied or expressed) obtained upon purchase of the item generally covers the early failures and the warranty period typically ends prior to the wearout period.

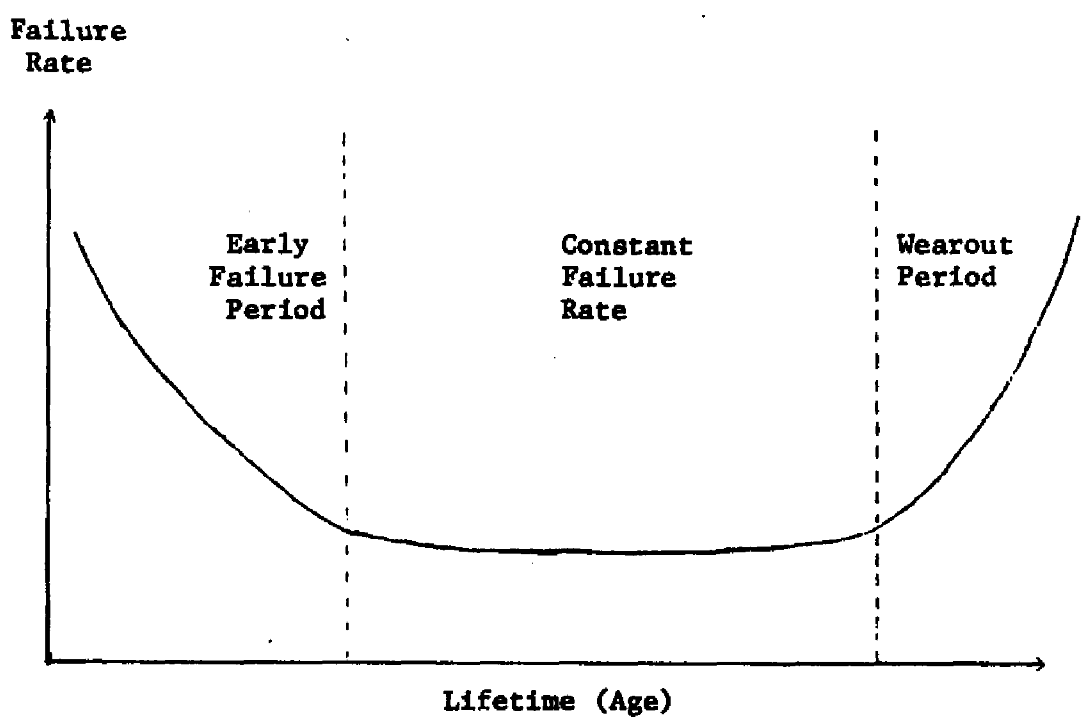

FIGURE 3

Population Failure Rate versus Age (Bathtub Curve)

To simulate the constant failure rate portion of the bathtub curve the Poisson distribution is used. This distribution leads to a simple negative exponential expression for the probability of a failure occurring as a function of time. The basic assumptions necessary for the use of the Poisson distribution are (1) that time may be divided into sufficiently small intervals so that the probability of two events in one interval of time is zero; and (2) that the probability of one event in any interval is a constant times the length of the interval and is independent of the other intervals. These assumptions seem reasonable in the context of this study.

The reciprocal of the failure rate is the mean time between failures. Thus if π is the constant failure rate, then the MTBF is $1/\pi$. Throughout most of the study, MTBF will be used rather than the failure rate since it is the more convenient in expressing the reliability of a device.

LIFE CYCLE COST--THE NON-RIW PROCUREMENT

The total Life Cycle Cost for a non-RIW procurement is the sum of all quantifiable costs associated with the procurement over a specific period of time. In order to compare the total Life Cycle Costs for the non-RIW and the RIW procurement, the time period of the warranty will be used as the period of measurement of costs in both cases. It is assumed that the length of the warranty period chosen for the RIW contract is such that it is less than the lifetime of the procured equipment.

The three major cost elements considered in the definition of the total Life Cycle Cost for the non-RIW procurement are the cost of the acquisition of the equipment, the direct costs with respect to maintenance of failures and the indirect costs associated with maintenance support. Total Life Cycle Cost is a function of the length of the warranty period. Because of this all cost components of the total Life Cycle Cost which represent an investment of owner dollars over time must be amortized to provide a fair comparison with respect to varying warranty periods. In this study investment dollars will be amortized on a straight-line basis.

Given below is the total Life Cycle Cost model for the non-RIW procurement alternative:

$$LCC_1(T_W) = U_1PA_M + C_{DM1} + C_{IS1}A_M + C_{RS1}T_W \qquad (1)$$

where

$LCC_1(T_W)$ = total life cycle cost measured over $(0,T_W)$ for a non-RIW procurement,

T_W = warranty period in months,

U_1 = number of units purchased,

P = purchase price of each unit,

A_M = amortization factor = T_W/T_L,

T_L = lifetime of equipment in months,

C_{DM1} = direct maintenance cost to the purchaser during the warranty period,

C_{IS1} = initial support cost to the purchaser,

and

C_{RS1} = recurring maintenance support costs per month.

Details of each component in the function LCC_1 are given in the sections which follow.

Number of Units Procured

For either procurement alternative the number of units procured is the sum of the number of operational units and the number of spares that will be required. Let U represent the number of units procured, U_o be the number of operational units and U_s be the number of spares. Then $U = U_o + U_s$.

There are many factors required in determining the number of spares U_s to be purchased. For example, some of the factors are the design of the pipeline used in repairing failures, the failure rate, the rate of repair and the geographic distribution of units among military bases. To simplify this matter the number of spares will be computed in terms of the number of items in the repair system. This value may be found by applying the following basic queueing theory formula for a single server system with Poisson failures.[2]

$$E(Q) = E(n)T_p \tag{2}$$

where

$E(Q)$ = expected number in the repair facility,

$E(n)$ = expected number of failures per month,

and

T_p = expected time each failure spends in the system measured in months.

Now we can determine the expected number of failures per month by $E(n) = (U_o H_o)/\overline{\theta}_e$ where U_o = the number of operational units,

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[2] Taha, Hamdy A., Operations Research: An Introduction, (New York: The MacMillian Co., 1971), p. 507.

H_o = the average number of hours a unit operates in a month and $\bar{\theta}_e$ = the estimated average MTBF of a unit over the warranty period.

Next assign to T_p the average time spent by each failure in the pipeline measured in months. Hence, the equation for defining the number of spares required for an average level of service may be written as follows:

$$U_s = E(Q)$$

$$U_s = E(n)\ T_p$$

$$U_s = \frac{U_o H_o T_p}{\bar{\theta}_e} \tag{3}$$

For the purchase without a RIW contract it is assumed that the MTBF is equal to its initial reliability, θ_i, throughout the warranty period. If $\bar{\theta}_e$ is replaced with θ_i in equation (3) the total number of spares required for an average level of service for the non-RIW case is

$$U_{S1} = (U_o H_o T_p)/\theta_i \tag{4}$$

Thus the total number of units procured in this case is

$$U_1 = U_o + (U_o H_o T_p)/\theta_i \tag{5}$$

<u>Total Direct Maintenance Cost</u>

In the case where the item is purchased without a RIW contract the owner incurs all the cost of failures. These costs are reflected in the total direct maintenance cost C_{DM1} which represents the sum of all direct costs associated with repairing failed equipment. These costs include expenses relative to the removal/replacement, shipping, testing and repairing of failed items.

The total direct maintenance cost C_{DM1} can be estimated by multiplying the expected number of failures over the warranty period times the average cost per failure. The equation below defines C_{DM1} as:

$$C_{DM1} = \text{(Expected Number of Failures)} \times \text{(Average Cost per Failure)}$$

$$= (U_o H_o T_W / \theta_i)\ C_{F1}$$

$$= (U_o C_{F1} H_o T_W) / \theta_i \qquad (6)$$

where

U_o = the number of operational units,

θ_i = initial or constant MTBF over the warranty period,

C_{F1} = cost per failed unit,

H_o = average number of hours a unit operates in a month,

and

T_W = length of the RIW contract in months.

Support Costs

As in the case of most procurements the purchaser incurs both initial and support costs in the non-RIW case. The initial support cost C_{IS1} includes the cost of test equipment, manuals, training, etc. The monthly recurring support cost C_{RS1} includes items such as administrative costs and preventative maintenance costs. Since the initial support cost represents an investment it is given as an amortized item in equation (1).

LIFE CYCLE COST--THE RIW PROCUREMENT

The major cost components of the total Life Cycle Cost for the procurement of equipment with a RIW contract are the initial acquisition costs of equipment, the costs of the warranty coverage, the direct costs associated with failures, and the indirect costs relative to maintenance support. Given below is the total life cycle cost model for the RIW procurement alternative.

$$LCC_2(T_W) = U_2PA_M + C_{RIW} + C_{DM2} + C_{IS2}A_M + C_{RS2}T_W \qquad (7)$$

where

$LCC_2(T_W)$ = total life cycle cost measured over $(0, T_W)$ for a procurement with a RIW contract,

T_W = warranty period in months,

U_2 = number of units purchased,

P = purchase price of each unit,

A_M = amortization factor = T_W/T_L

T_L = lifetime of equipment in months,

C_{RIW} = Total cost of RIW contract,

C_{DM2} = direct maintenance support cost to the owner,

C_{IS2} = initial support cost to the owner,

and

C_{RS2} = recurring maintenance support costs per month.

Details of each component of the function LCC_2 are given in the sections that follow.

<u>Number of Units Procured</u>

The number of units U_2 purchased by the military in a RIW procurement is $U_2 = U_o + U_{S2}$, where U_o = the required number of operational units and U_{S2} = the number of spares. An estimate of $\overline{\theta}_e$, the average MTBF of a unit during the warranty period, is required to determine the number of spares U_{S2}. It is known that the MTBF of the equipment has an initial value of θ_i and each contract is negotiated to motivate the contractor to bring the equipment up to at least a specified MTBF of θ^* by the end of the warranty period. It is thus reasonable to define $\theta_e = (\theta_i + \theta^*)/2$. By using this estimate of $\overline{\theta}_e$ and equation (3) we have

$$U_{S2} = (U_o H_o T_p)/\overline{\theta}_e \tag{8}$$

where

U_o = the number of operational units,

H_o = average number of hours a unit operates per month,

and

T_p = average time each failure spends in the repair pipeline measured in months.

Thus the value of U_2 becomes

$$U_2 = U_o + (U_o H_o T_p)/\overline{\theta}_e \tag{9}$$

<u>Cost of the RIW Contract</u>

As mentioned in Chapter 1 each RIW contract involves a fixed price which must be negotiated between the user and the contractor. The negotiated price includes the expected direct maintenance cost of the contractor. The expected direct maintenance cost of the contractor is dependent upon the anticipated number of failures during the warranty period and the cost of each repair action. Once the expected direct maintenance cost of the contractor and the cost of equipment modifications resulting from the reliability program have been agreed upon, the contractor applies a risk factor and a profit factor to arrive at the contract price.

The contractor faces risk and uncertainty when he engages into an RIW contract. The risk and uncertainty increases as the length of the warranty increases. One risk function utilized by contractors to compensate for this fact is

$$R(T_W) = (1 + r)^{T_W/12} \tag{10}$$

where

r is the annual rate of risk.[3]

Given below is an equation for estimating the total cost of employing a RIW contract for U_2 items for the warranty period.

$$C_{RIW} = \{C_{MOD} + C_{DMC}\}R(T_W)\ (1 + X/100) \tag{11}$$

where

C_{RIW} = total cost of the RIW contract,

C_{MOD} = total expected modification cost of the contractor,

$R(T_W)$= risk factor,

and

X = percent profit of the contractor

The subtopics which follow develop the models to determine the total expected modification cost C_{MOD} and the total direct maintenance cost to the contractor C_{DMC}.

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[3]Balaban, Harold S. and Bernard L. Retterer, "The Use of Warranties for Defense Avionics Procurement," The ARINC Research Corporation Report 0637-02-1-243, The ARINC Research Corporation, Annapolis, Maryland, June, 1973, p. 101.

<u>The effect of equipment modifications on MTBF</u>. Modifications of equipment under a RIW are made through the use of engineering change proposals. Each engineering change proposal is designed to improve the current MTBF by a factor of M. Namely, if θ_{OLD} is the current MTBF, then the new MTBF $\theta_{NEW} = M \times \theta_{OLD}$. Within the lifetime of a RIW contract a finite number of modifications may be implemented to cause the initial MTBF to approach a specified MTBF, which was agreed upon in the contract negotiations. Thus a technique must be found to determine the improvement factor M defined by each modification.

The value of each M is bounded such that $M \geq 1$ but $M < M'$, where M' is an upper bound for all such modifications pertaining to the particular unit. Furthermore the value of each M is dependent upon θ^* the specified MTBF the contractor must try to reach with each modification and upon θ the current MTBF of the population of items procured.

One such function that is consistent with these assumptions is

$$M(\theta) = M' + (1 - M')\left(\frac{M'-M^*}{M'-1}\right)^{\frac{\theta^*}{\theta}} \tag{12}$$

where

M^* = the improvement factor expected if $\theta = \theta^*$.

This function is a form of the Pearl-Reed curve often used in economic growth models. Balaban and Retterer[4] used a similar form of

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[4]<u>Ibid</u>., p. 93.

this equation in their study. A sketch of the modification function M is given below:

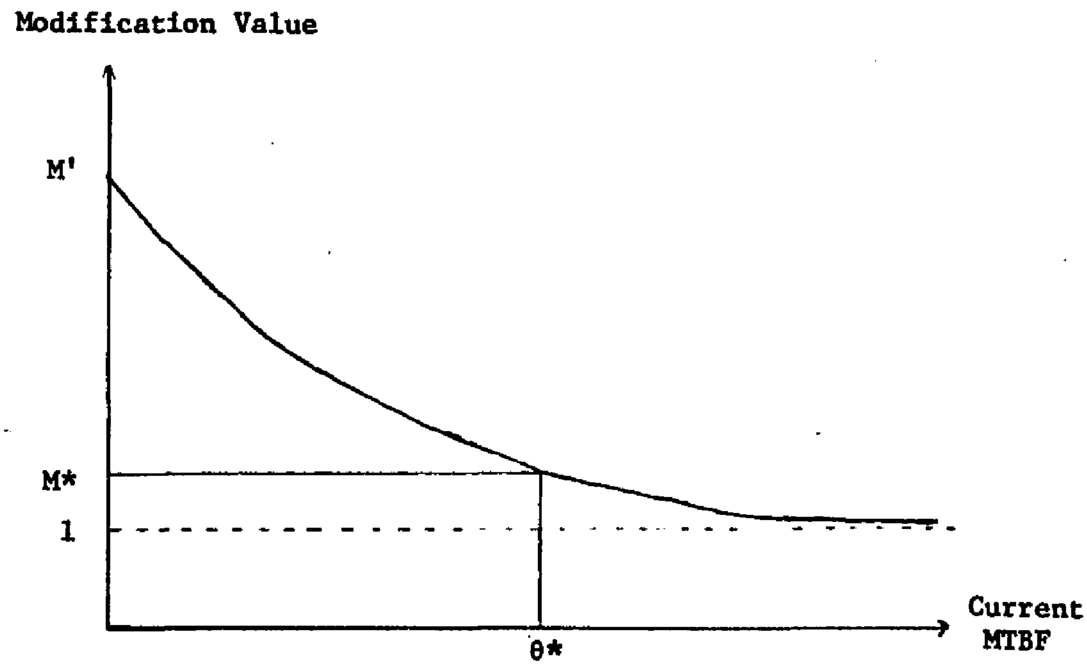

Figure 4

Modification Value versus Current MTBF

It should be noted that $\lim_{\theta\to\infty} M(\theta) = 1$ since $\frac{\theta^*}{\theta} \to 0$ as $\theta \to \infty$. Also $\lim_{\theta\to 0} M(\theta) = M'$ since $0 < \left(\frac{M'-M^*}{M'-1}\right) < 1$ and $\frac{\theta^*}{\theta} \to \infty$ as $\theta \to 0$ which implies that $\left(\frac{M'-M^*}{M'-1}\right)^{\frac{\theta^*}{\theta}} \to 0$. Furthermore it is seen that $M(\theta^*) = M^*$.

A numerical example is considered to demonstrate the nature of the function M. Let the initial MTBF be 50 hours, the specified MTBF be 111 hours, $M' = 4$ and $M^* = 1.3$. For these values the modification function is

$$M(\theta) = 4 + (1-4)\left(\frac{4-1.3}{4-1}\right)^{\frac{111}{\theta}} = 4 - 3(.9)^{\frac{111}{\theta}}$$

Table 1 that follows indicates the values of M and the new MTBF θ_{NEW} after each modification is employed. Two modifications are required for θ_{NEW} to become greater than or equal to the specified MTBF of 111 hours.

Table 1

Modification Functional Values

Modification	θ	$M(\theta)$	$\theta_{NEW} = M x \theta$
1	50.0000	1.6257	81.2839
2	81.2839	1.4020	113.9818

<u>Time of modification</u>. The time at which a modification can be introduced is a random variable T_m. It is reasonable to assume that modifications will not occur before some minimum time T_α has occurred after procurement of the units or after a previous modification has been employed. One distribution that can be used to define T_m is the negative exponential distribution defined by $f(T_m) = de^{-d(T_m - T_\alpha)}$ for $T_m > T_\alpha$. The constant d represents the rate of modification.

The cumulative distribution function F is defined as

$$F(T) = P(T_m \leq T) = 1 - e^{-d(T_m - T_\alpha)} \text{ for } T_\alpha < T_m \leq T. \qquad (13)$$

Assume that a certain procurement results in n-modifications with values $M_1, M_2, \ldots, M_n$. Each modification occurs at some time T_m. The times between modifications are restricted in that each modification occurs only after some time T_{α} has occurred. Thus if n-modifications are involved then the times of each are ordered as

$$0 < T_{\alpha_1} < T_{m_1} < T_{\alpha_2} < T_{m_2} < \ldots < T_{\alpha_n} < T_{m_n} < T_w.$$

Given that a modification occurs on some time interval (T_{α}, T_{β}). then the expected time of such a modification can be obtained by employing basic principles of probability theory.

$$E(T_m | T_{\alpha} < T_m < T_{\beta}) = \frac{\int_{T_{\alpha}}^{T_{\beta}} T_m f(T_m) dT_m}{F(T_{\beta}) - F(T_{\alpha})} .$$

$$= \int_{T_{\alpha}}^{T_{\beta}} T_m \; f(T_m) dT_m \div (1 - e^{-d(T_{\beta} - T_{\alpha})}).$$

$$= T + \frac{1}{d} - \frac{(T_{\beta} - T_{\alpha}) \; e^{-d(T_{\beta} - T_{\alpha})}}{1 - e^{-d(T_{\beta} - T_{\alpha})}} . \tag{14}$$

Let $\overline{T}_m$ denote the expected modification time given by equation (14). The formula above will be employed on several occasions in the development of the model.

Average MTBF over T_w. The average MTBF of the equipment over the warranty period represents another important measure. If we assume that n-modifications occur respectively, at times T_{m_1}, T_{m_2}, ..., T_{m_n}, then the MTBF varies from θ_0 over $\left[0, T_{m_1}\right]$, to θ_1 over $\left[T_{m_1}, T_{m_2}\right]$, to θ_2 over $\left[T_{m_2}, T_{m_3}\right]$, ..., to θ_n over $\left[T_{m_n}, T_w\right]$. The values of the modification times T_{m_1}, T_{m_2}, ..., T_{m_n} can be estimated by employing equation (14) which defines $\overline{T}_m$.

Thus an estimate for the average MTBF $\bar{\theta}$ is defined as

$$\bar{\theta} = \frac{\theta_o(\bar{T}_{m_1})}{T_w} + \frac{\theta_1(\bar{T}_{m_2} - \bar{T}_{m_1})}{T_w} + \ldots + \frac{\theta_n(T_w - \bar{T}_{m_n})}{T_w} \tag{15}$$

Cost of modifications. This is a difficult cost to predict. Studies have shown that in general the greater the reliability improvement the higher the cost. Certainly there are instances where high reliability improvement has resulted from a low cost modification while a low reliability improvement has resulted from a high cost of modification. In general the cost of modification is an increasing function of M and this assumption will be used.

A study by Mercurio and Skaggs[5] utilized multiple regression analysis to obtain the cost of reliability improvement in terms of the resultant MTBF and the quantity of parts in the item modified. Balaban and Retterer[6] in their study utilized a cost function in terms of the item modified. The cost function adopted in this study is essentially the same as Balaban and Retterer's except the cost is defined in terms of the amount of modification to increase the MTBF of the item. The function is denoted and defined as

$$C(M) = 1.06(\exp\left[(M-1)/10M\right] - 1)P \tag{16}$$

where

P = the purchase price of the item

and $1 \leq M < M'$.

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[5]Mercurio, Salvotore P. and Clyde W. Skaggs, "Reliability Acquisition Cost Study," General Electric Company Report RADC-TR-73-334, Rome Air Development Center, Griffin Air Force Base, New York, November 1973, p. 27.

[6]Balaban and Retterer, op. cit., p. 95.

Presented below are the costs associated with the modifications defined previously in Table 1. Assume that P = \$10,000.

Table 2

Cost of Modifications

Modification	M	C(M)
1	1.6257	\$415.93
2	1.4020	\$308.34

In general the function C has a graph as illustrated in Figure 5.

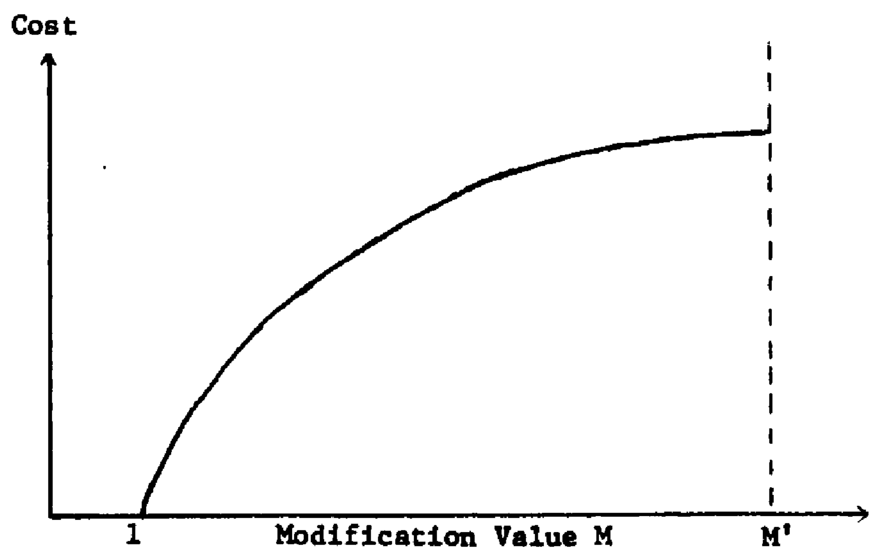

Figure 5

Cost of Modification versus Modification Value

<u>Contractor modification strategy</u>. The strategy utilized by the contractor in determining whether a modification to improve reliability

is to be attempted during the warranty period is next considered. It is assumed that the contractor implements a modification of equipment if it is profitable to do so.[7]

The decision rule is to establish the modification if the cost to produce the modification is less than the saving established from increasing the current MTBF from θ to $M_i\theta$ or reducing the failure rate from π to π/M_i. The saving obtained from the modification M_i is determined by finding the difference between the contractor cost of maintenance for the current failure rate π and the improved failure rate π/M_i over the period of time from T_{m_i} to T_w.

The contractor cost of maintenance with a failure rate π over the interval (T_{m_i}, T_w) is

$$C_1 = C_F U_0 \pi (T_w - T_{m_i}) H_0 \tag{17}$$

where

C_F = the cost per failure,

U_o = the number of operational units,

π = the current failure rate in failures/hour,

H_o = the number of operational hours of the equipment/month,

T_w = warranty period in months,

and

T_{m_i} = time of modification in months.

If at time T_{m_i} the contractor makes the appropriate equipment modification the failure rate reduces to π/M_i while the cost of

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[7]Balaban and Retterer, op. cit., p. 97.

failures over the period T_{m_i} to T_w becomes

$$C_2 = C_F U_o (\pi/M_i)(T_w - T_{m_i})\, H_o \tag{18}$$

Thus the expected savings is

$$C_1 - C_2 = C_F U_o (1 - 1/M_i)\pi(T_w - T_{m_i})\, H_o \tag{19}$$

The contractor's strategy is to apply the modification at time T_{m_i} if it is profitable. Namely, the modification of value M_i is applied if its total cost is less than or equal to the savings it produces. If $C(M_i)$ represents the cost of modifying one unit, then the total cost of modifying U_2 units is $U_2 x C(M_i)$. The inequality below represents the decision rule to use when applying a modification.

$$U_2 C(M_i) \leq C_F U_o (1 - \frac{1}{M_i})\, \pi(T_w - T_{m_i}) H_o \tag{20}$$

The above inequality is important because it will lead us to a time T_i to act as an upper bound for T_{m_i}. Solving the inequality for T_{m_i} yields

$$T_{m_i} \leq T_w - \frac{U_2 C(M_i)}{U_o (1 - 1/M_i)\pi C_F H_o} \tag{21}$$

Defining

$$T_i = T_w - \frac{U_2 C(M_i)}{U_o (1 - 1/M_i)\pi C_F H_o} \tag{22}$$

produces an upper bound for T_{m_i}. Since the two preceding inequalities are logically equivalent, then we can apply the contractor's decision rule equivalently by noting whether or not the value obtained for T_i is bigger than the minimum time to the next modification T_{α_i}. The latter is true because T_{m_i} exists provided $T_{\alpha_i} < T_{m_i} \leqslant T_i$. Namely, it is profitable to apply the modification with value M_i if

$T_{\alpha_i} < T_i$ since it implies the existence of a T_{m_i} between T_{α_i} and T_i.

<u>Algorithm to generate $\overline{T}_{m_i}$ values.</u> Assume it is known that the k-modifications are required to improve the reliability of the equipment such that the MTBF of the equipment is greater than or equal to the specified MTBF of the contract. It must be decided whether the modifications are profitable.

Let T_{min} represent the minimum amount of time that must elapse prior to the first modification or the amount of time that must elapse between successive modifications. A flowchart of the basic algorithm for determining the expected modification times for k potential modifications is found in figure 6.

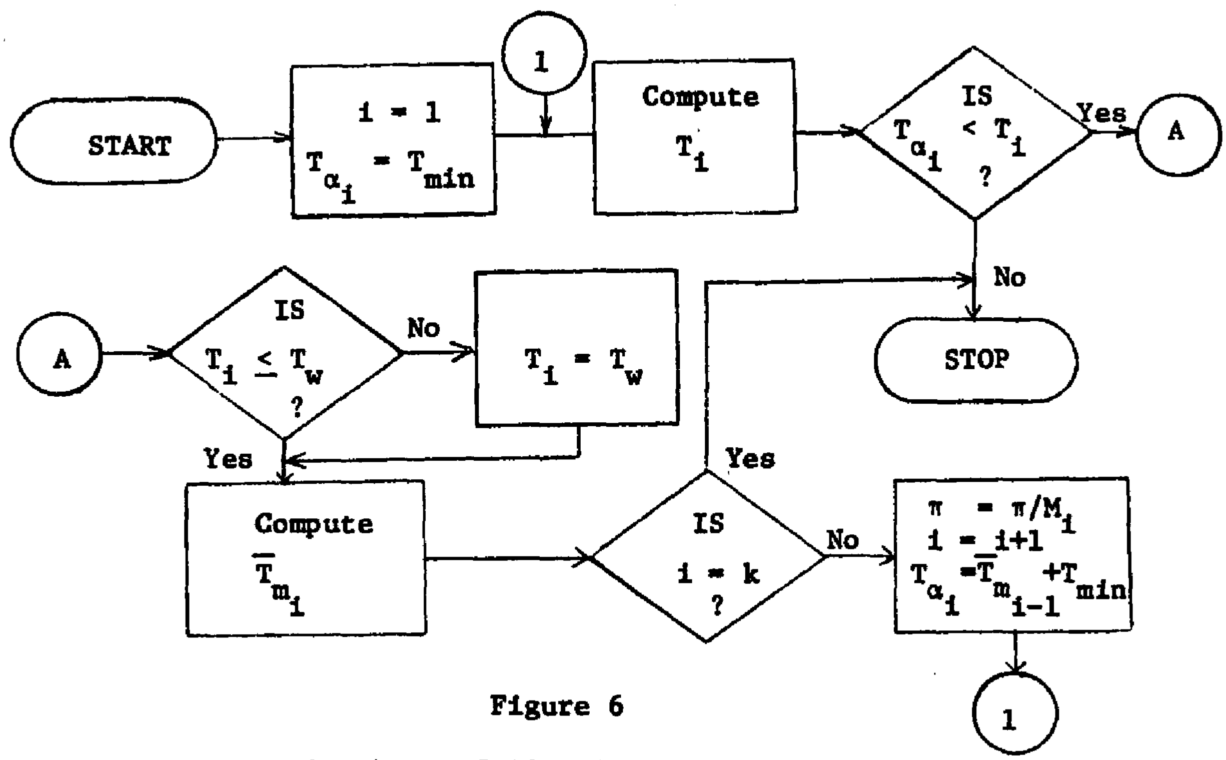

Figure 6

Flowchart of Algorithm to Compute

Expected Modification Times

Assume an RIW contract is in effect. It is now helpful to follow the logic of the Algorithm. The value of i is set to 1. Next T_1 is computed with π equal to the initial failure rate of the item. The value of T_1 is compared to T_{α_1}, to determine if a modification of value M_1 is profitable.

If $T_{\alpha_1} < T_1$, then the modification is profitable. Since T_1 must fall within the warranty period we must insure that $T_1 \leq T_w$. If it is found that $T_{\alpha_1} < T_1$, but $T_1 > T_w$, then let $T_1 = T_w$. Thus we find in either case $T_{\alpha_1} < T_1 \leq T_w$. By employing the formula for the expected modification time the value of $\overline{T}_{m_1}$, can be found. The algorithm continues until $i = k$.

If $T_{\alpha_1} \geq T_1$, then the modification is deemed unprofitable and no other modifications are considered during the course of the warranty period.

Table 3 that follows indicates the corresponding values of $\overline{T}_{m_i}$ for the values of M_i found in Table 1. For the determination of the entries in Table 3 assume $T_{min} = 3$ months, $T_w = 60$ months, $T_L = 120$ months, $U_2 = 1040$ units, $U_o = 1000$ units, $H_o = 25$ hours/ month, $C_F =$ \$120 per failure and $d = .070$.

Table 3

Expected Modification Times

Modification	M_i	T_{α_i}	T_i	T_{m_i}
1	1.6257	3.00	41.27	14.47
2	1.4020	17.47	29.70	22.72

<u>Total expected cost of modification</u>. If it is found that k-modifications are profitable, then the total cost of these modifications can be found by employing the formula

$$C_{MOD} = U \sum_{i=1}^{k} C(M_i) \tag{22}$$

The contractor typically will incur modification costs at time T_{m_i}. If we assume the estimated modification costs are included by the contractor in the price of the RIW contract and the military pays the contract price at time 0, the contractor normally discounts these costs. The formula that follows determines the total cost of all the modifications by discounting and amortizing each $C(M_i)$

$$C_{MOD} = U \sum_{i=1}^{k} C(M_i) \frac{1}{(1 + I/12)^{T_{m_i}}} \left(\frac{T_w - T_{m_i}}{T_L - T_{m_i}} \right) \tag{23}$$

where

I = yearly interest rate.

In the formula the modification times T_{m_i} are estimated by expected modification times $\overline{T}_{m_i}$.

<u>Contractor direct maintenance cost</u>. The contractor incurs a total direct maintenance cost C_{DMC} because the terms of the RIW contract make him responsible for failures. This cost is determined by applying the same formula utilized to compute the user direct maintenance cost C_{DM1} with the cost per failure C_F designating the contractor's cost per failure and θ set equal to the expected MTBF over the warranty period. In general the efficiency of the contractor will bring about a smaller cost per failure than that obtained by the user in the non-RIW case.

Support Costs

There are three support costs encountered by the user of equipment warranted with a RIW contract. These three include the user's direct maintenance cost, initial support cost, and recurring support cost.

The military incurs a direct maintenance cost whether the given equipment is under a RIW contract or not. If a RIW contract is utilized, then the military has only the expenses relative to removal/replacement of the failed unit along with the cost of shipping the unit to the contractor. The value of the total user direct maintenance cost C_{DM2} for a RIW contract can be determined by multiplying the expected number of failures by the average cost per failure. This yields the equation below.

$$\begin{aligned} C_{DM2} &= (U_o H_o T_w / \bar{\theta}) C_{F2} \\ &= (U_o C_{F2} H_o T_w) / \bar{\theta} \qquad (24) \end{aligned}$$

where

U_o = the number of operational units,

C_{F2} = cost per failed unit,

H_o = average number of hours a unit operates in a month,

T_w = length of warranty period in months,

and

$\bar{\theta}$ = the average MTBF over the warranty as found by equation (15).

In general the initial support cost C_{IS2} represents a saving for a RIW-procurement because a substantial investment in test and support equipment, manuals and other start-up costs is not required. The amount of saving is dependent upon whether the item is one already in the military's inventory or not. If the item is in the military's inventory, then the saving is smaller because an initial support cost has already been incurred. Since C_{IS2} represents an investment it is amortized on a straight line basis.

Recurring support cost C_{RS2}, which includes such items as administrative and preventative maintenance costs, may be higher than its non-RIW counterpart. This is due to the fact that the cost of administering the RIW is more involved.

ASSESSING THE ECONOMIC VALUE OF A RIW

Although a RIW offers a potential benefit to the military user, it is not applicable in all procurements. The model developed in previous sections of this chapter determines whether or not a warranty is economically attractive by computing the total amortized saving (loss) of a RIW for a period of T_w months, i.e.,

$$\text{SAVING } (T_w) = LCC_1(T_w) - LCC_2(T_w) \tag{25}$$

If the RIW procurement is economically attractive, i.e., a saving for some time period of T_w months exists, the model determines the optimum number of months that the contract should be employed. This is carried out by testing all integer values of T_w from 12 months to 120 months.[8] The lower bound of 12 months is used to test the model

[8] The economic value of a RIW contract model has been programmed in Fortran IV. A detail flowchart for this program is found in Appendix A.

because the nature of a RIW requires a sufficiently long period for the reliability improvement program to be conducted. The upper bound of 120 months is used because it represents the average lifetime of avionic equipment.

To illustrate the model, five hypothetical procurements will be used. Values for each of the cases have been extrapolated from data obtained from existing RIW contracts within the Department of Defense. It is felt that these sample procurements represent a cross-section of equipment type, complexity, reliability and costs.

Assumptions

The parameters given in table 4 will be used for the evaluation of the five hypothetical procurements.

Table 4

Parameters for Economic Value Model

Parameter	Symbol	Value
Equipment Lifetime	T_L	120 months
Minimum Period Before Modification	T_{min}	3 months
Discount Interest Rate	I	10%
Risk Factor	r	4%
Contractor Profit Factor	x	10%
The Rate of Modification	d	.1096

Each of the values in Table 4 represents the average or expected value associated with equipment that might be subjected to a RIW contract. The value for the rate of modification d was derived by assuming that the probability that a contractor would inaugurate an equipment modification over the period 3 months to 24 months was .90. By employing equation (13) obtained for the cumulative distribution of the random variable T_m and the information of the last sentence, the value of the rate of modification d can be found.

The key values M* and M' which are required in the modification improvement function are defined as $M' = 4$ and $M^* = 1 + .2(\theta^* - \theta_i)/\theta_i$ where θ^* is the specified MTBF and θ_i is the initial MTBF. The value of M' and the defining relation for M* were extrapolated from data acquired from existing RIW contracts.

Input for the Hypothetical Procurements

Table 5 lists the corresponding data elements for hypothetical procurements A, B, C, D, and E. The unit price of the five equipment types vary from relatively inexpensive to expensive. Reliability for the five cases ranges from a low-MTBF unit to a high-MTBF unit. Quantities to be purchased among the five cases vary from a small purchase size to a large purchase size. Also variations in the initial support costs of the five items indicate procurements of items already in inventory and items which are new and are not currently in the military's inventory.

Results

Table 6 presents the results obtained by applying the economic value of a RIW model to the five hypothetical procurement cases.

Table 5

Data Base for Test Cases

	Symbol	Proc. A	Proc. B	Proc. C	Proc. D	Proc. E
Unit Price	P	19,461.80	15,461.80	23,091.80	516.30	1,842.70
Operating Hours per Month	H_o	31.4	52.0	82.2	36.5	68.1
Number of Operational Units	U_o	435	27	858	1,023	168
Initial MTBF	θ_i	1,002.3	312.4	288.0	103.3	200.1
Specified MTBF	θ^*	1,157.5	472.3	430.0	131.1	269.1
User Cost per Failed Unit Non-RIW	C_{F1}	13,049.16	6,111.78	16,878.50	327.64	151.28
User Cost per Failed Unit with RIW	C_{F2}	1,288.27	608.41	1,663.55	41.56	24.28
Contractor Cost per failed Unit	C_F	12,142.66	5,391.49	14,889.33	226.53	116.21
Pipeline Time of Failed Unit	T_P	1.87	2.11	2.28	1.95	1.49
Initial Support Cost Non-RIW	C_{IS1}	50,000.	200,000.	60,000	20,000.	370,000.
Initial Support Cost with RIW	C_{IS2}	10,000.	0	0	0	31,000.
Recurring Support Cost Non-RIW	C_{RS1}	500	500	800	1,500	500
Recurring Support Cost with RIW	C_{RS2}	800	800	1,200	2,400	800

Table 6

Summary of Pertinent Output from Model

	Symbol	Proc. A	Proc. B	Proc. C	Proc. D	Proc. E
Optimum Warranty Period (Months)	T_w	12	75	63	59	38
Cost of RIW per Unit	--	4,848.09	53,644.81	77,474.30	3,442.96	1,132.33
Average MTBF Over Contract Period	$\bar{\theta}$	1,024	428	379	118	233
Final MTBF	FMTBF	1,038	505	458	131	274
Units Procured Non-RIW Case	U_1	460	36	1,416	1,728	253
Units Procured with RIW	U_2	459	35	1,305	1,644	241
Total Life Cycle Cost Non-RIW	$TLCC_1$	3,032,247	2,570,453	277,648,227	7,524,406	517,305
Total Life Cycle Cost with RIW	$TLCC_2$	3,326,759	2,425,326	271,633,490	6,996,600	499,077
Saving(Loss)	--	-294,512	145,127	6,014,737	527,806	18,227

Procurement A which represents a moderately large purchase of a high-MTBF unit with a moderately high purchase price was the only one of the five procurements found not to be acceptable. Its optimum saving(loss) was found to be - $294,512 for a period of 12 months.

Figures 7 through 11 display the warranty saving(loss) as a function of the warranty period. The graphs of the saving functions for procurements B, C, D, and E reflect definite optimum values for each procurement.

The results of the contractor reliability improvement programs for procurements B, C, D, and E are summarized in Table 7. Each of these procurements required four modifications and in each case the final MTBF exceeded the specified MTBF.

Table 7

Summary of Modification Programs

Variable	Proc. B	Proc. C	Proc. D	Proc. E
Number of Modifications	4	4	4	4
Initial MTBF	312.4	288.0	103.3	200.1
Specified MTBF	472.3	430.0	131.3	269.1
Average MTBF	428.0	379.0	118.0	233.0
Final MTBF	505.0	458.2	131.5	274.2
Optimum Warranty Period	75	63	59	38

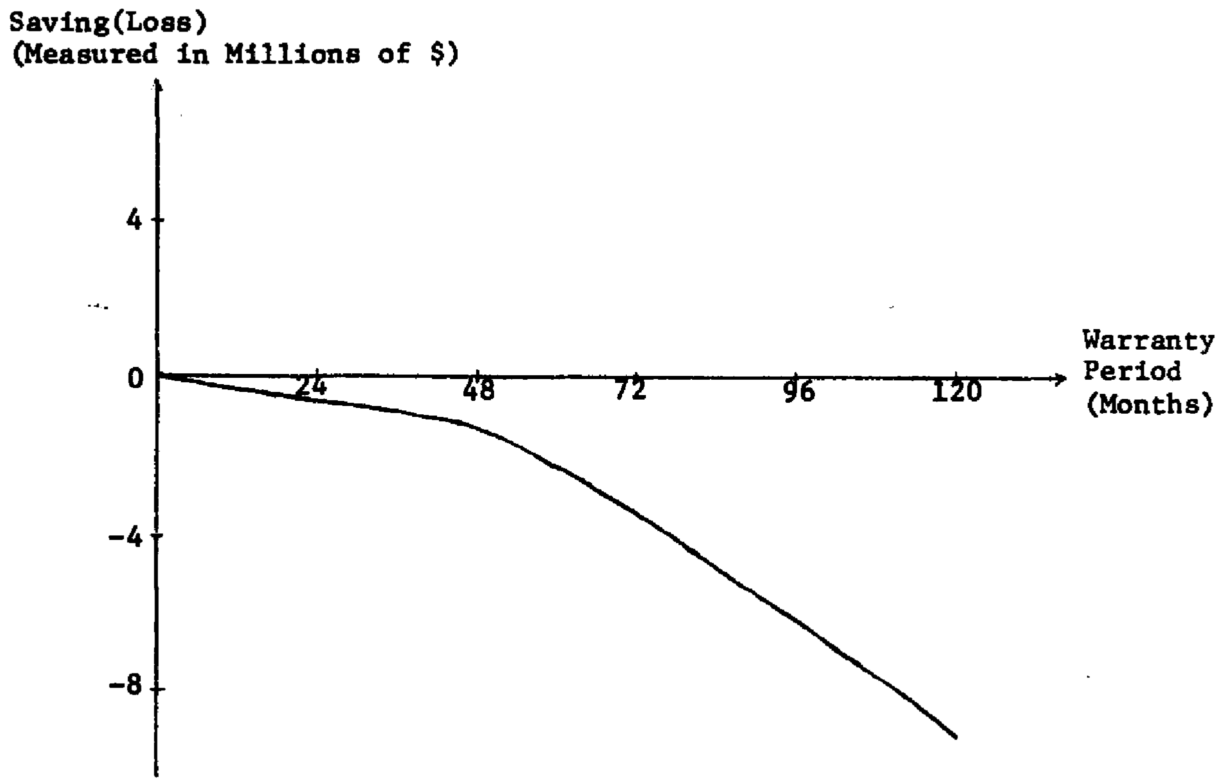

Figure 7

Saving(Loss) vs. Warranty Period

Procurement A

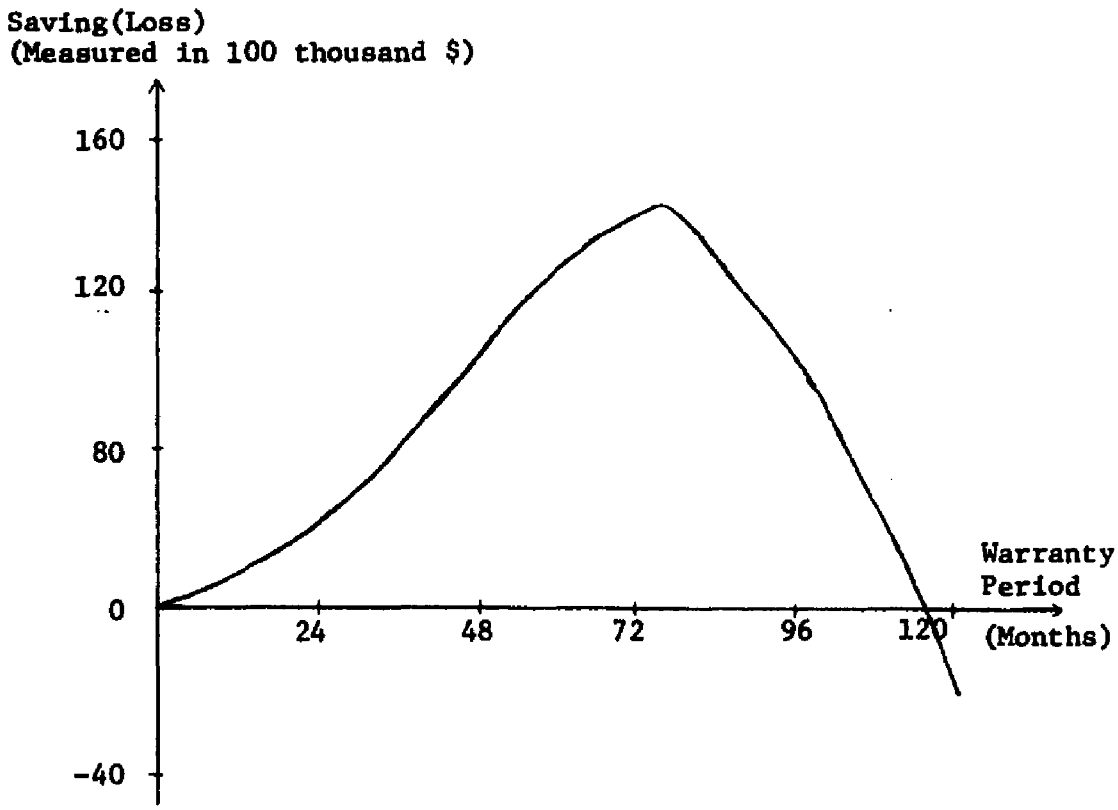

Figure 8

Saving(Loss) vs. Warranty Period

Procurement B

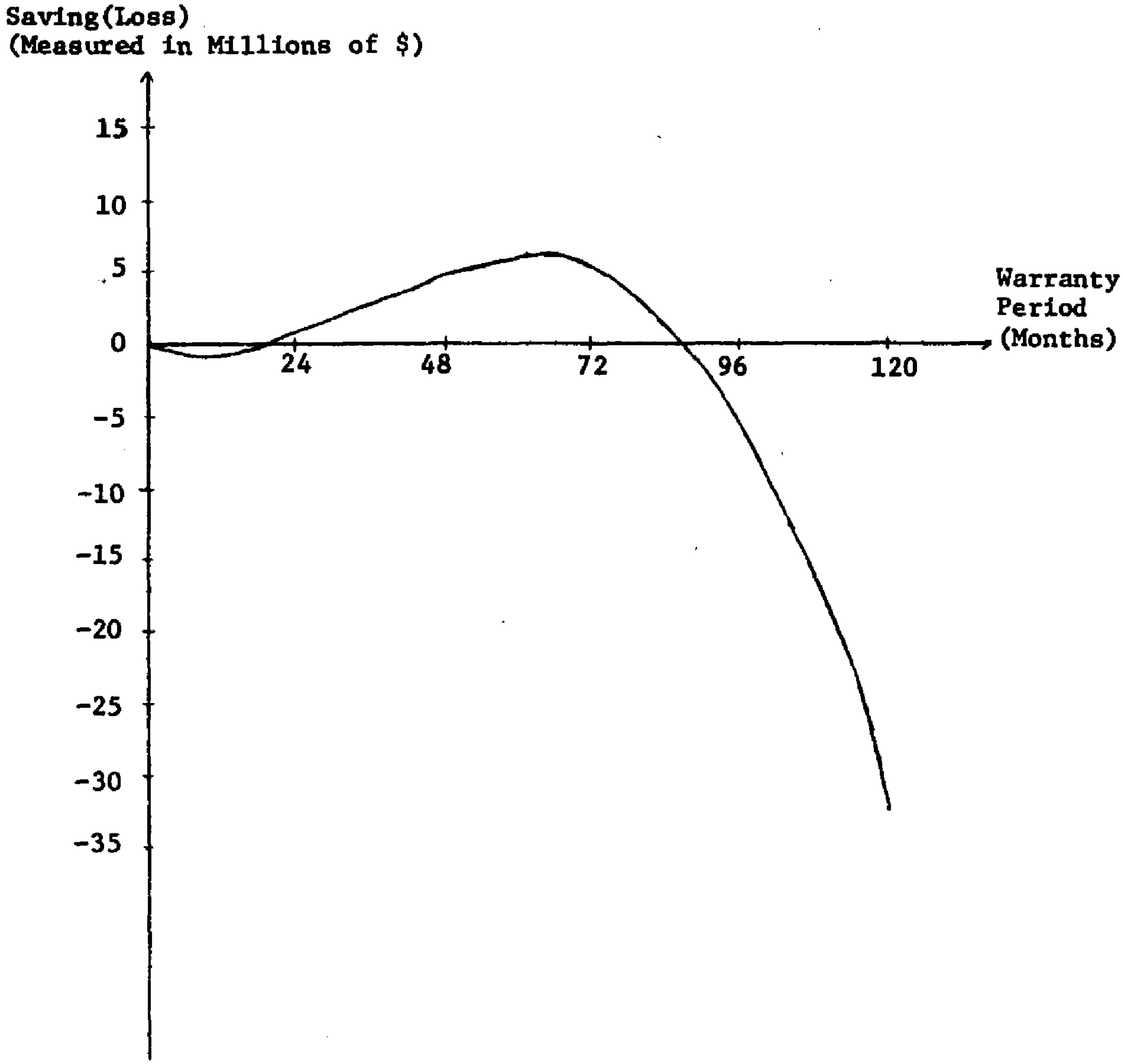

Figure 9

Saving(Loss) vs. Warranty Period

Procurement C

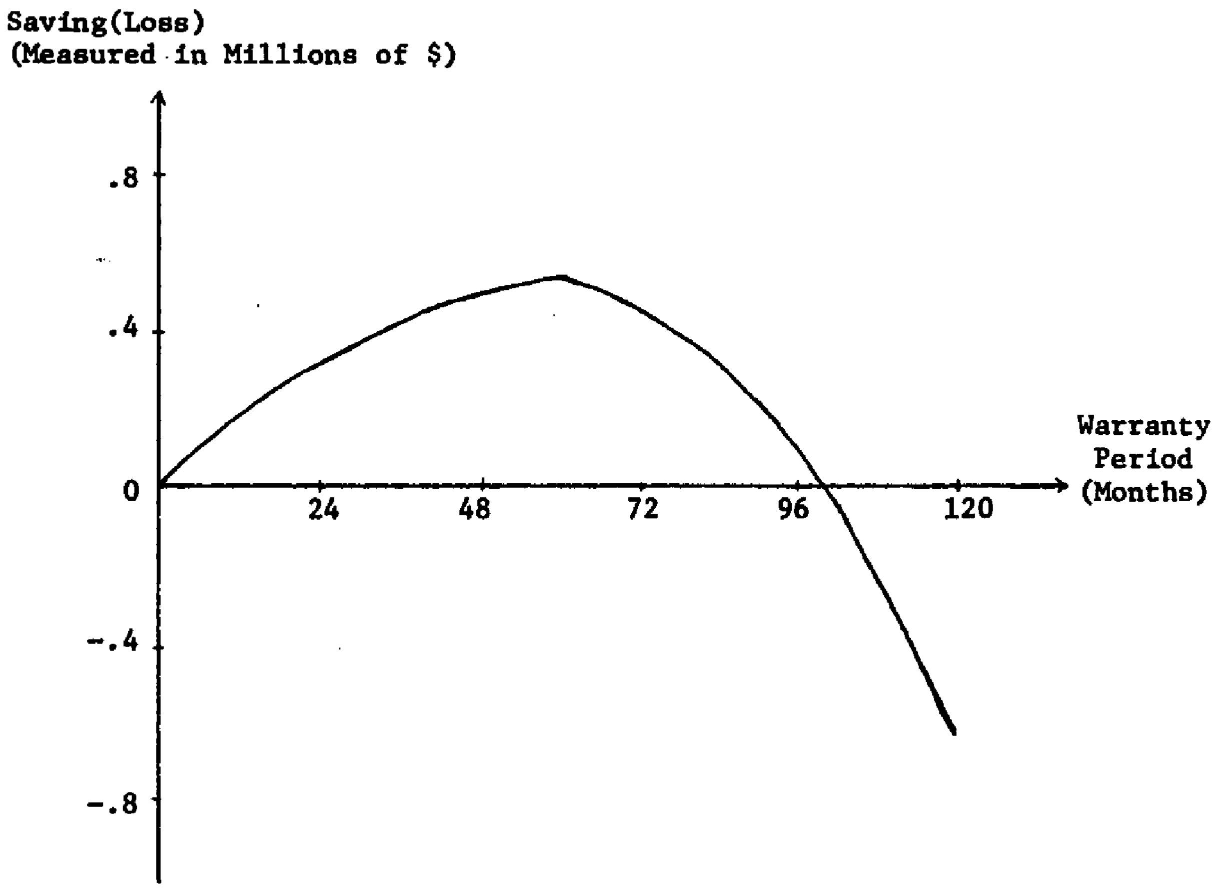

Figure 10

Saving(Loss) vs. Warranty Period

Procurement D

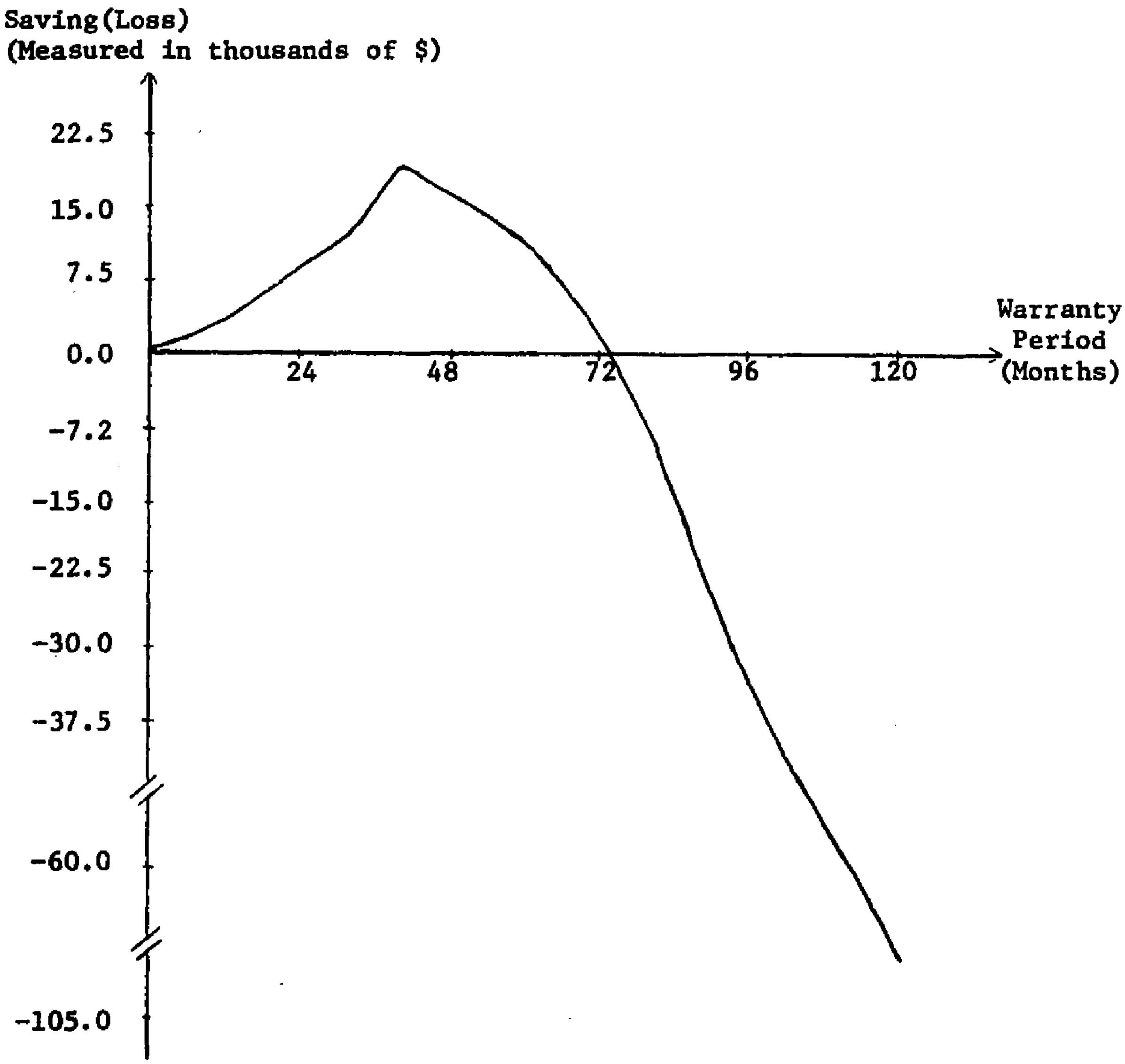

Figure 11

Saving(Loss) vs. Warranty Period

Procurement E

CHAPTER 3

DISCRIMINANT ANALYSIS

In many situations it is necessary to classify or assign an individual to two or more groups under conditions of uncertainty. For example, it may be necessary to decide whether a college applicant is suitable to enter a particular college, or whether a firm should extend open-book credit to all new credit applicants for a sample period. Analogously in this study it is necessary to decide whether or not a procurement should be made with or without a RIW contract.

The method of two-way discriminant analysis is employed to classify a procurement into one of two groups--a non-RIW procurement group or a RIW procurement group. A requirement in applying the technique of discriminant analysis is that a set of measures be taken with respect to specific variables which are commonly defined on the elements of the two groups. The theory of this classification technique is dependent upon the set of means taken on the measures of each group together with the corresponding variances and covariances of the measures of both groups. Fundamentally the discriminant model classifies an arbitrary procurement to the particular group whose characteristics are similar to its own.

TWO-WAY DISCRIMINANT ANALYSIS

In order to implement the methodology of two-way discriminant analysis it is necessary to define a finite number of variables on the elements of the non-RIW procurement group and on the elements of the RIW procurement group. To illustrate the steps in employing the model let us assume for the purpose of simplicity that the classification of items into the two groups is dependent upon only two variables--say X and Y.

The first step in the analysis is to estimate the coefficients C_x and C_y for the linear discriminant function f defined below:

$$f(X,Y) = C_x X + C_y Y \tag{1}$$

To estimate the coefficients of the function f, samples of size n_1 and n_2 must be respectively drawn from the non-RIW and the RIW groups. Once the values of the coefficients are found a value of f can be calculated for any combination of X and Y, whether the combination is from the original samples or a new sample.[1]

Next a critical value of f called a cut-point is determined by utilizing the functional values generated by the function f for the samples of the two groups. If an unclassified procurement has a functional value determined by f that is greater than or equal to this cut-point, the procurement is classified into the non-RIW group. If, however, the corresponding functional value defined by f for the unclassified procurement is less than the break-point, then the procurement is classified in the RIW group. It is because of this classification technique that the name "discriminant analysis" comes

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[1]Appendix B contains the details of how to determine the coefficients of f.

forth. That is, the function f is defined so that it discriminates between the two groups of procurements.

The goal of discriminant analysis is to assign an unclassified procurement to one of the two procurement groups in such a way that the probability of misclassification is minimized. Figure 12 that follows presents a graphic illustration of the basic notions of discriminant analysis. In the figure the letter A indicates the non-RIW group and the letter B indicates the RIW group. The scatter of points obtained from the samples from each population is enclosed by an ellipse which reflects some specified protion of the corresponding population. Let L be a line drawn through the points where the two ellipses intersect. Now consider a third axis which is drawn from the origin so that it is perpendicular to the line L. The coefficients of the function f are defined in such a way that the function f projects the points of both populations to this third axis. Thus the function f maps the two multivariate populations of procurements into two univariate populations. It can be seen that the overlap between the univariate populations A' and B' (represented by the shaded area) is smaller than would be obtained by any other line drawn through the ellipses formed by the scatter plot. Hence the probability of misclassification is minimized since the area of overlap is minimized.

The values f_1 and f_2 in Figure 12 represent the average values of the functional values defined by f with respect to the two

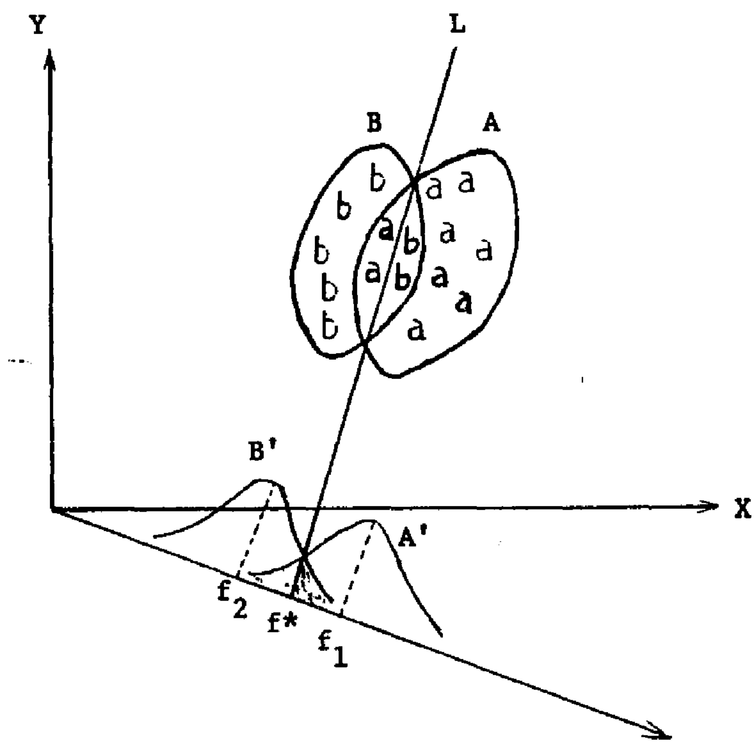

Figure 12

Illustration of the Discriminant Function

procurement groups. The figure indicates that f_2 is sufficiently smaller than f_1. However, there is an area of overlap between the populations. The smaller the area of overlap, the greater the probability of the discriminant function to correctly classify the non-RIW and RIW procurements. As mentioned the coefficients for the discriminant function are determined in such a way to minimize the size of the area of overlap or to maximize the proportion of procurements that will be classified correctly. The f* value in Figure 12 defines the break-point. If a value of f for an unclassified procurement is larger than or equal to f*, then the procurement is classified into the non-RIW group. Otherwise the procurement is classified as being applicable for RIW.

As mentioned we considered only two variables to view measures from the two populations of procurements so that the initial discussion of two-way discriminant analysis would be easier to understand. This discussion can be extended to involve p-variables to reflect measures for both classes of procurements, where $P \geq 2$. The fundamental difference is that the linear discriminant function f is defined as a function of p-variables rather than one of two variables. Thus the general model requires the estimation of p-coefficients to represent f.

From a geometric standpoint the two populations would be distributed by p-variates rather than two variates. However, the function f would still project the two classes of procurements into two univaraite distributions as was illustrated by the previous figure.

The computational scheme developed in Appendix B is based upon several assumptions about the two classes of procurements. Namely, both populations of procurements must be normally distributed with equal covariance matrices. If large sample sizes are available many researchers tend to ignore the issue of validating these assumptions on the grounds that the classification procedure is fairly robust. The assumptions for obtaining unbiased estimates of the coefficients of f have not been formally worked out for discriminant analysis.

However, it is quite likely that the assumptions are similar to those for regression analysis.[2]

A MONTE CARLO GENERATOR OF PROCUREMENTS

One of the benefits the electronic digital computer has given us is the ability to compile huge amounts of experience which reflect the behavior of certain variables under controlled conditions. In those cases where a random number generator is used as the source for generating this data the activity is called a Monte Carlo study.[3] As mentioned earlier it is necessary that a sample of hypothetical procurements, which have been classified into the non-RIW and RIW procurement groups, be defined to represent a data base for the discriminant analysis model. Before the sample can be drawn, the universe of potential RIW procurements must be described in terms of a set of variables and their corresponding probability distributions. It is these variables and their distributions that form the working mechanism of the Monte Carlo generator of procurements.

Once the universe of procurements has been described in terms of the Monte Carlo generator, a sample of 200 sets of measures with each set representing a procurement will be generated by using the

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[2]Frank, Ronald E. and Paul E. Green, Quantitative Methods in Marketing (Englewood Cliffs: Prentice Hall, 1967), p. 74.

[3]Cooley, William W. and Paul R. Lohnes, Multivariate Data Analysis (New York: John Wiley, 1971), p. 270.

model.[4] Elements of the sample will then be classified into one of the two procurement groups through the application of the economic value of a RIW contract model of the previous chapter. The data base obtained from this step is to be used to employ the discriminant analysis model.

Elements of the universe of potential candidates for application of a RIW contract will be described in terms of 9 variables. These key variables were chosen because they reflect the necessary measures required to employ the economic value of a RIW model. The table which follows lists these variables and their notation. It is assumed that the universe of potential procurements for RIW application are items already within the military's inventory. Because of this, the initial support cost will not be defined since its effect would be negligible. Also the monthly recurring support cost discussed in the model of Chapter 2 will be omitted since its effect is also in general negligible in the determination of the economic value of a RIW contract.

Two sources of information were used to find the necessary parameters to define the distributions of the key variables of the following table. One source of information was obtained from a survey of past and present RIW contracts within the Department of Defense. The results of the survey were not as complete as desired because most of the RIW contracts written by the Government are still active. Therefore, no final conclusion could be drawn about the distributions of certain key variables.

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[4]The listing of a Fortran IV program which represents the model is found in Appendix C.

Table 8

Key Variables of the Procurement Generator

Variable	Description
U_0	Number of Operating Units
H_0	Expected number of operating hours/month
θ_i	Initial MTBF
θ^*	Specified MTBF
P	Unit Price
C_{F1}	User Cost per failed unit without RIW
C_{F2}	User Cost per failed unit with RIW
C_F	Contractor cost per failed unit
T_P	Time in failure pipeline in months

The second source of information came from a study conducted by the Hughes Aircraft Company on the F-106A aircraft.[5] This study of the F-106A aircraft revealed reliability and cost of failure data on the inventory of front line units of this fighter plane. The cost information was adjusted to reflect the rate of inflation over the years since the report.

It should be pointed out that the distributions that will be defined later for the key variables of Table 8 in no way reflect the

[5] R.H. Myers and others, "Airborne Electronic Equipment Lifetime Guarantee," Hughes Aircraft Company, Rome Air Development Center Technical Report 69-363, November, 1969, pp. 22-56.

universe of all procurements made within the Department of Defense. However, it is felt that the distributions to be defined represent characteristics of a subcollection of avionic items that the procurement officer would most likely experience for possible RIW application.

Distributions for U_0 and H_0

The number of operational units associated with a procurement and the expected number of hours of operation of the equipment are defined in view of data obtained from the survey of RIW contracts within the Department of Defense. Operational units for a procurement will be defined by employing an exponential distribution with a mean of 380 units. Values generated by the exponential random number generator will be half adjusted to the nearest whole unit.

The expected number of operating hours per month of a particular equipment type will be described by a normal distribution with a mean of 56 hours and a standard deviation of 16.4 hours.

Distributions for θ_i, θ^* and T_p

The initial MTBF for an item is defined by the discrete distribution defined in Table 9 that follows. The source of information used to define this specific distribution was obtained from the reliability information gathered on the F-106A aircraft.[6]

[6] *Ibid.*, pp. 226-26.

Table 9

Distribution of the Initial MTBF of a Procurement

a	b	$\text{Prob}(a \leq \theta_i < b)$
51 hrs.	1,000 hrs.	.786
1,000 hrs.	2,000 hrs.	.092
2,000 hrs.	6,000 hrs.	.069
6,000 hrs.	16,400 hrs.	.053

The specified MTBF θ^*, which reflects the reliability improvement goal of the contractor is defined as $\theta^* = x\ \theta_i$, where x is an improvement factor which is uniformly distributed between 1.02 and 1.50. The factor x produces a potential improvement in reliability of 2% to 50% over the contract period.

Pipeline time, which reflects the time a failed unit remains in a state of failure, is defined to be uniformly distributed between .5 and 3.25 months. Both the distribution for the specified MTBF and the pipeline time were obtained from the data compiled on existing contracts.

Distribution of C_{F1}, C_{F2}, C_F and P

User cost per failure for an item not under a RIW is defined in terms of the unit price of the item. Namely,

$$C_{F1} = yP \tag{2}$$

where

y = a factor which is uniformly distributed between .05 and 1.05,

and

P = the unit price of the item.

Cost per failure in this case can be as low as 5% of the unit price and as high as 105% of the unit price.

User cost per failure for an item under a RIW is expressed as a linear function of the user cost per failed unit without a RIW contract. Namely,

$$C_{F2} = 9.452 + .098\ C_{F1} \tag{3}$$

This relation was obtained by applying the technique of regression analysis to data obtained from the survey of RIW contracts within the Department of Defense. The coefficient of correlation between the two variables was .93 for the sample of values.

As in the case of the initial MTBF, the unit price P is defined by a discrete distribution. The data base for this distribution was obtained from the Hughes study of the F-106A aircraft.[7]

Contractor cost per failed unit is defined in terms of the user cost per failure for the non-RIW procurement case. In particular it is expressed as

$$C_F = qC_{F1}$$

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[7] *Ibid.*, pp. 52-56.

Table 10

Distribution of Unit Prices of Procurements

a	b	Prob ($a \leq P < b$)
\$ 200	\$ 4,015	.750
4,015	7,830	.131
7,380	11,645	.025
11,645	15,460	.020
15,460	19,275	.018
19,275	23,090	.019
23,090	26,905	.025
26,905	30,720	.012

where

q = a factor which is uniformly distributed between .6 and .9. It is assumed that because of the contractor's efficiency at repairing failed units, his cost per repair will be 60% to 90% of what it normally would cost the military.

CLASSIFICATION INTO SAMPLE GROUPS

After the sample of 200 procurements has been generated by the Monte Carlo generator, each element must be classified into either the non-RIW or the RIW procurement groups. This task is carried out by applying the economic value of an RIW model of Chapter 2 to each element of the sample. Application of the economic value of the RIW model to the sample of 200 designated 58 procurements into the non-RIW group and 142 into the RIW group. The number of elements in each group was expected because many of the characteristics of the Monte Carlo generator were based on past RIW contracts. In Appendix D there are two tables which respectively list the non-RIW group and RIW group of procurements in terms of their key measures.

APPLICATION OF THE DISCRIMINANT MODEL

Given the two groups of procurements determined in the last section it is now time to apply the discriminant model. The results that were obtained for the two groups of procurements defined in the last section were found by applying the computer program BMD04M developed by the Department of Biomathematics at the University of California at Los Angeles.[8]

In applying the technique of discriminant analysis it is prudent to choose the combination of explanatory variables that will produce the best discriminant function. After trying various combinations of variables, six variables were selected to define the linear discriminant function. Table 11 which follows on the next page presents the explanatory variables aong with their corresponding coefficients.

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[8]Dixon, W.J., ed. BMD: Biomedical Computer Programs, (Los Angeles: University of California Press, 1974), pp. 221-220.

Table 11

Coefficients of Explanatory Variables

Variable	Description	Coefficient
1	(total number of units purchased) x .01	.00026
2[a]	(number of failures per month) x .1	-.00013
3	(initial MTBF) x .01	.00010
4	relative change of the initial MTBF to the specified MTBF	-.03963
5[b]	(user monthly maintenance cost) x .00001	-.00014
6	ratio of user cost per failed unit to the unit price	-.00313

[a]Number of failures per month is defined as $(U_0H_0)/\theta_i$.

[b]User monthly maintenance cost is defined as $(H_0U_0C_{F1})/\theta_i$.

Variables 1, 2, 3, and 5 in Table 11 were scaled because their corresponding entries in the covariance matrix were so large in their natural form that exponential overflows occurred when the inverse of the covariance matrix was determined by the BMD04M program.

CONFUSION MATRIX

Since it is known beforehand which group each sample procurement actually belongs to, we can prepare a table of correct and incorrect classifications as made by the discriminant function. This score sheet of correct and incorrect classifications is referred to as a confusion matrix. From the confusion matrix the success of the discriminant function can be discovered. Namely, the fewer the misclassifications of procurements into the two procurement groups, then the better the model.

The results of applying the discriminant function to the values of the six explanatory variables for the 200 sample procurements are found in Appendix E. The average functional value for the 58 elements in the non-RIW group is $f_1 = -.00473$ while the average functional value for the 142 elements in the RIW group is $f_2 = -.01235$. The cut-point for the entire sample of 200 is $f^* = -.00854$, which is the average of f_1 and f_2.

The criteria for classification is that those procurements with functional values greater than or equal to the cut-point are classified into the non-RIW group while those procurements with functional values less than the cut-point are classified into group 2. Using this criteria the confusion matrix that follows was determined for the sample of 200 procurements.

Table 12

Confusion Matrix for Sample

Actual Membership	Predicted Group Membership Non-RIW	RIW	Total
Non-RIW	40	18	58
RIW	41	101	142

The totals along the diagonal of the matrix indicate the number of correct classifications or hits for the given discriminant function. There were 141 hits which indicates 70.5% of the procurements were classified correctly.

Discriminatory Power

It is now of interest to determine the discriminatory power of the function obtained in this application. To test the discriminatory power of the discriminant function a chi-square test is applied.[9] Accordingly, it is necessary to find

$$Q = (n-e)^2/e + (\bar{n}-\bar{e})^2/\bar{e} \tag{5}$$

where n and $\bar{n}$, respectively, denote the correct and incorrect clasifications made by the function and e and $\bar{e}$ respectively denote the expected number of classifications if classifications were made at random.

Using the fact that the probability of a successful random classification is .5, then $e = \bar{e} = 100$ for the sample of 200. From Table 12 it is found that $n = 141$ and $\bar{n} = 59$. Applying the definition of Q it is found that $Q = 33.62$.

Our objective is to test the following hypothesis:

H_0: Correct classifications occurred randomly.

H_A: The discriminant function did better than chance.

The variable Q is chi-square distributed with one degree of freedom. At the one percent level it is found from a standard table that chi-square

[9]Press, S. James, Applied Multivariate Analysis, (New York: Holt, Rinehart, and Winston, Inc., 1972), pp. 382-383.

with one degree of freedom is 10.8. Thus it follows that Q = 33.62 is certainly significant and we reject the null hypothesis. Hence we conclude that the discriminant function does better than chance.

An Interval Cut-Point

In some cases an interval cut-point is used as a basis for the classification rather than a single point. Examining the table of functional values for the sample of 200 procurements, which is found in Appendix E, it is seen that the values are ordered according to their f values. Furthermore there is an area of overlap of the two procurement groups which falls between the 6th ranked value of f which is +.00436 and the 163rd ranked value of f which is -.01722. When referring to these values, we will denote b = -.01722 and c = +.00436.

If we have reason to believe that any procurement under examination for a RIW contract will not differ significantly from the relationships found for the sample procurements, then the interval with endpoints b and c can be used in the following way: Unclassified procurements with f values less than or equal to b are accepted for RIW contracts while those procurements with f values greater than or equal to c are placed in the non-RIW category. In those cases where unclassified procurements have f values falling between points b and c additional information is required before a decision is made.

CHAPTER 4

CONCLUSIONS AND RECOMMENDATIONS

The objective of this study is to extend the methodologies presently available to the United States Air Force with respect to the application of the Reliability Improvement Warranty concept. Two models are developed to quantify the criteria presently used for the selection of RIW contracts.

The first model deals with the determination of the economic value of a RIW contract. Development of this model was contingent upon the understanding of the RIW concept. Computing the economic value of a RIW contract involves finding the difference between the Life Cycle Cost for the non-RIW alternative and the RIW alternative. Only costs that varied with the warranty's terms and conditions were considered in the development of the model.

The second model utilizes the theory of discriminant analysis to judge whether or not to employ a RIW in a procurement. This model requires defining a linear function in terms of a group of key variables, which represented measures defined upon the universe of procurements. The first step in employing the model involves calculating the coefficients of the terms of the linear function. A requirement for the computation of the set of coefficients is that samples be available from both the non-RIW and RIW procurement groups.

Due to the limited experience of the Department of Defense with RIW contracts a very small data base was available to carry out this phase of the study. To overcome this obstacle, the Monte Carlo technique is utilized to generate the required data base. Each component of the generated procurement is defined in terms of a set of characteristic values which includes cost and reliability measures. Members of the sample were later classified into non-RIW or RIW groups by employing the economic value of the RIW model. Given the two groups of sample procurements, the coefficients of the linear function were found. Then the discriminant analysis technique was applied accordingly.

CONCLUSIONS

Enumerated below are the conclusions drawn from this study to extend the methodology of when to employ a RIW contract in making a procurement. Each conclusion is based solely on the models employed in this study and the assumptions under which they operate.

1. This study leads to three possible approaches to determining when to employ a RIW contract.
    a. Approach number one is to apply the economic value of a RIW contract to classify a procurement to either a non-RIW or RIW group.
    b. Given a representative data base for both procurement groups, then approach number two involves utilizing a discriminant analysis model to classify procurement types.

c. Approach number three is a combination of the first two approaches. Given the appropriate conditions for discriminant analysis application, an interval cut-point is found. If an arbitrary procurement has a discriminant functional value greater than or equal to the right-endpoint of the interval, it is classified into the non-RIW group. Furthermore, if the arbitrary procurement has a discriminant functional value that is less than the left-endpoint of the interval, then it is classified into the RIW group. In those cases where the procurement has a functional value which falls between both endpoints of the interval, then the economic value of the RIW model should be applied. This approach is better than approach number two in that it reduces the risk of misclassification induced by discriminant analysis.

2. Based upon the limited success of the RIW concept within the Department of Defense and in certain areas of the civilian sector, the number of applications of the RIW contract will probably increase sharply. The three approaches given in the first conclusion are independent of the avionic industry and thus can be used by buyers in other areas.

3. The economic value of a RIW model gives the procurement officer a method of determining the destiny of a procurement relative to a RIW application. Like any model of a

complex process it does not include all factors. However, it does provide estimates of various costs and reliability variables associated with the warranty.

4. In those cases where a decision to employ a RIW has been made, the economic value model can be employed to find the optimal or most cost-effective warranty period.
5. Even though the economic value of a RIW model determines the optimal warranty period, the warranty period should be long enough to permit the benefits of the warranty to be accomplished. Proper use of the RIW requires the product to reach a state of design stability so that specified reliability improvements can be employed. A rule of thumb might be to use a warranty period of at least three years.
6. The economic value of a RIW model simplifies the task of warranty pricing. Based upon cost and reliability estimates, the cost of a RIW may be obtained. Hence, for the procurement officer, the pricing of a RIW contract is not any more difficult to find than the determination of the appropriate number of spares required in any purchase.
7. The cut-point developed in the application of the discriminant model of Chapter 3 is the equivalent of the break-even point for the application of RIW contracts.

RECOMMENDATIONS

A number of recommendations are made in this section with regard to the future extension of the methodology of when an RIW should be applied.

1. One of the limitations of the economic value of a RIW model is that it does not treat the influence of inflation upon Life Cycle Cost. This model should be extended to include the effects of inflation.
2. Another limitation of the economic value of a RIW model is that it restricts the determination of the Life Cycle Cost in both alternatives to the given warranty period. It is recommended that the economic value model be extended to include the option to renew or extend a RIW beyond its contract period. Certainly there are situations when the option to renew or extend a RIW contract beyond its initial period may be attractive. For example, in those programs requiring a large investment in initial support costs, the option to renew the RIW contract should be a valuable alternative to have in the initial contract. At the same time it is good to leave the final agreement to extend or renew a RIW contract open until the initial warranty period is near its completion.
3. The economic value model should be extended to determine both the optimal warranty period and the optimal specified MTBF which maximize the saving due to a RIW contract.

4. The economic value model should be extended to compare the Life Cycle Cost related to the RIW alternative to other warranty alternatives such as the newly developed RIW with Guaranteed MTBF coverage considered on the F-16 fighter plane.
5. The submodel of the economic value model which determines the number of spares for a procurement should be extended to cover various circumstances such as the geographic location of the repair depot with respect to the units under warranty and the variations in the structure of the failure pipelines.
6. The design of a common data base for the collection of pertinent data with respect to all RIW contracts within the Department of Defense is necessary if future studies and evaluations are to be made on the RIW topic.
7. The efforts of Chapter 3 are considered only to be exploratory in nature. A full scale extension of the discriminant analysis model is recommended as a worthy undertaking, because it would equip the procurement officer with an efficient method of classifying a large inventory of products to either the RIW group or non-RIW group.

8. Discriminant analysis has been employed in this study as a means of classifying procurements into the non-RIW or RIW groups. It is recommended that the method of discriminant analysis be investigated by the Department of Defense for possible application in other areas as a tool in making decisions under uncertainty.

BIBLIOGRAPHY

Allen, Dennis Jean. "Application of the Reliability Improvement Warranty to DOD Procurements." Unpublished Master's thesis, Navy Postgraduate School, Monterey, California, March 1975.

American Law Institute and National Conference of Uniform State Laws. Uniform Commercial Code. West Publishing Company, 1972 Official Text.

Anderson, T.W. An Introduction to Multivariate Statistical Analysis. New York: John Wiley, 1958.

Balaban, Harold S. and Bernard L. Retterer. "The Use of Warranties for Defense Avionics Procurement." The ARINC Research Corporation Report 0637-02-1-243. ARINC Corporation, Annapolis, Maryland, 1973.

Bazovsky, Igor. Reliability Theory and Practice. Englewood Cliffs: Prentice Hall, 1961.

Campbell, William Giles and Stephen Vaughn Ballou. Form and Style: Theses, Reports, Term Papers. Fourth ed., Boston: Houghton Mifflin Co., 1974.

Cooley, William W. and Paul R. Lohnes. Multivariate Data Analysis, New York: John Wiley, 1971.

Dixon, W.J., ed. BMD: Biomedical Computer Programs. Third ed., Los Angeles: University of California Press, 1973.

Dunn, Payton E. and Andrew A. Oltyan. "Evaluation of Proposed Criteria to be Used in the Selection of Candidates for Reliability Improvement Warranties." Unpublished Master's thesis, School of Systems and Logistics, Air Force Institute of Technology (Air University), Wright-Patterson AFB, Ohio, 1975.

Frank, Ronald E. and Paul E. Green. Quantitative Methods in Marketing. Englewood Cliffs: Prentice Hall, 1967.

Klass, Philip J. "Failure Free Warranty Idea Lauded, Wider Use Desired," Aviation Week and Space Technology, (Feb. 9, 1970), pp. 57-58.

Logistics Management Institute. "Criteria for Evaluating Weapon System and Costs." LMI Task No. 73-11, Washington, D.C., March 1974.

Markowitz, Oscar. "Life Cycle Costing Applied to the Procurement of Aircraft Spare Parts." Unpublished Master's thesis, Drexel University, Philadelphia, Pa., 1971.

_______. "When Purchasing and Design Look to Total Costs," Purchasing Magazine, (June, 1972), pp. 65-69.

Massy, William F. "On Methods: Discriminant Analysis of Audience Characteristics," Journal of Advertising Research, vol. 5, (March, 1965), pp. 39-48.

Mercurio, Salvatore P. and Clyde W. Skaggs. "Reliability Acquisition Cost Study." RADC-TR-73-334, General Electric Company, November 1973.

Meyers, R.H. and others. "Airborne Electronic Equipment Lifetime Guarantee." Hughes Aircraft Company, Rome Air Development Center, Technical Report 69-363, 1969.

Nixon, Harvey L., Jr. and Christopher B. Hitchcock. "A Simulation of the Reparable Processing Procedures Applicable to Reliability Improvement Warranties." Unpublished Master's thesis, School of Systems and Logistics, Air Force Institute of Technology (Air University), Wright-Patterson AFB, Ohio, August 1975.

Press, S. James. Applied Multivariate Analysis. New York: Holt, Rinehart and Winston, Inc., 1972.

Taha, Hamdy A. Operations Research: An Introduction. New York: The MacMillian Company, 1971.

U.S. Department of the Air Force. Interim Guidelines: Reliability Improvement Warranty (RIW). Washington, D.C., 1974.

_______. Reliability Improvement Warranties Description and Use. Air Force Business Research Management Center. Wright-Patterson AFB, Ohio, 1974.

U.S. Department of the Navy. Proceedings at Failure Free Warranty Seminar, Dec. 12-13, 1973. U.S. Navy Aviation Supply Office, Philadelphia, Pennsylvania, 1973.

APPENDIX A

FLOWCHART OF ECONOMIC VALUE OF RIW MODEL

The mainline program for determining the economic value of a RIW contract is supported by 3 function subprograms and 3 subroutine subprograms. Given below is a list of these subprograms and a description of each.

MOD is a subroutine subprogram which determines the vector of modifications to increase the MTBF of an item from θ_i to θ^*.

COST is a subroutine subprogram which determines the corresponding vector of costs associated with each element of the modification vector returned by subroutine MOD.

TIME is a subroutine subprogram which computes a vector of expected times for the corresponding vector of feasible modifications returned by the subroutine MOD.

MTBF is a function subprogram which computes the final MTBF as a result of applying the vector of feasible modifications.

AMTBF is a function subprogram which determines the average or expected MTBF of the item over the contract period.

ECMOD is a function subprogram which determines the expected total cost of all feasible modifications implemented during the warranty period. The cost of each modification is discounted and amortized.

The figure which follows represents a general flowchart of the mainline FORTRAN IV program, which determines the warranty period that maximizes the saving for a RIW contract.

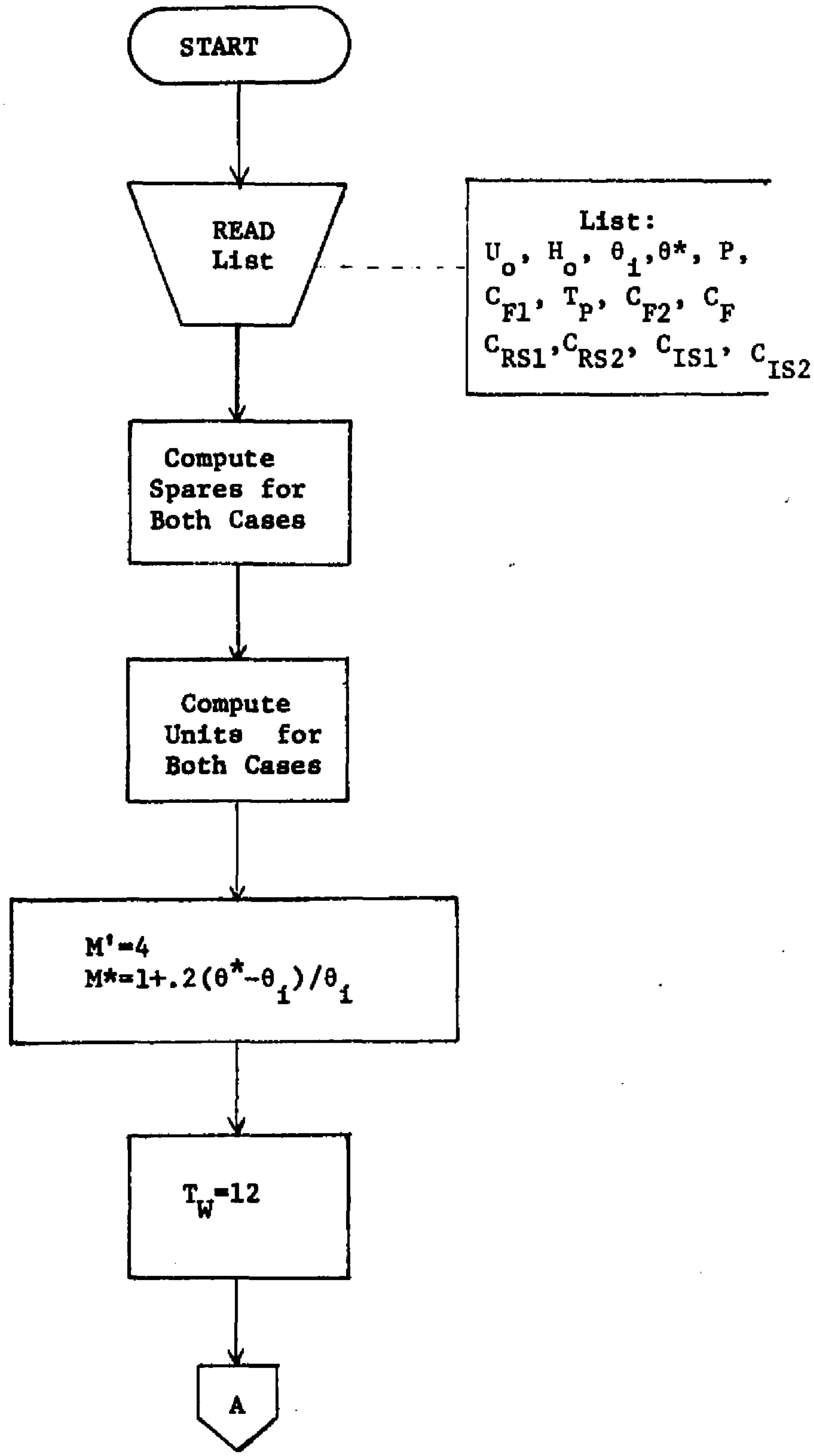

Figure 13

Economic Value of RIW Model

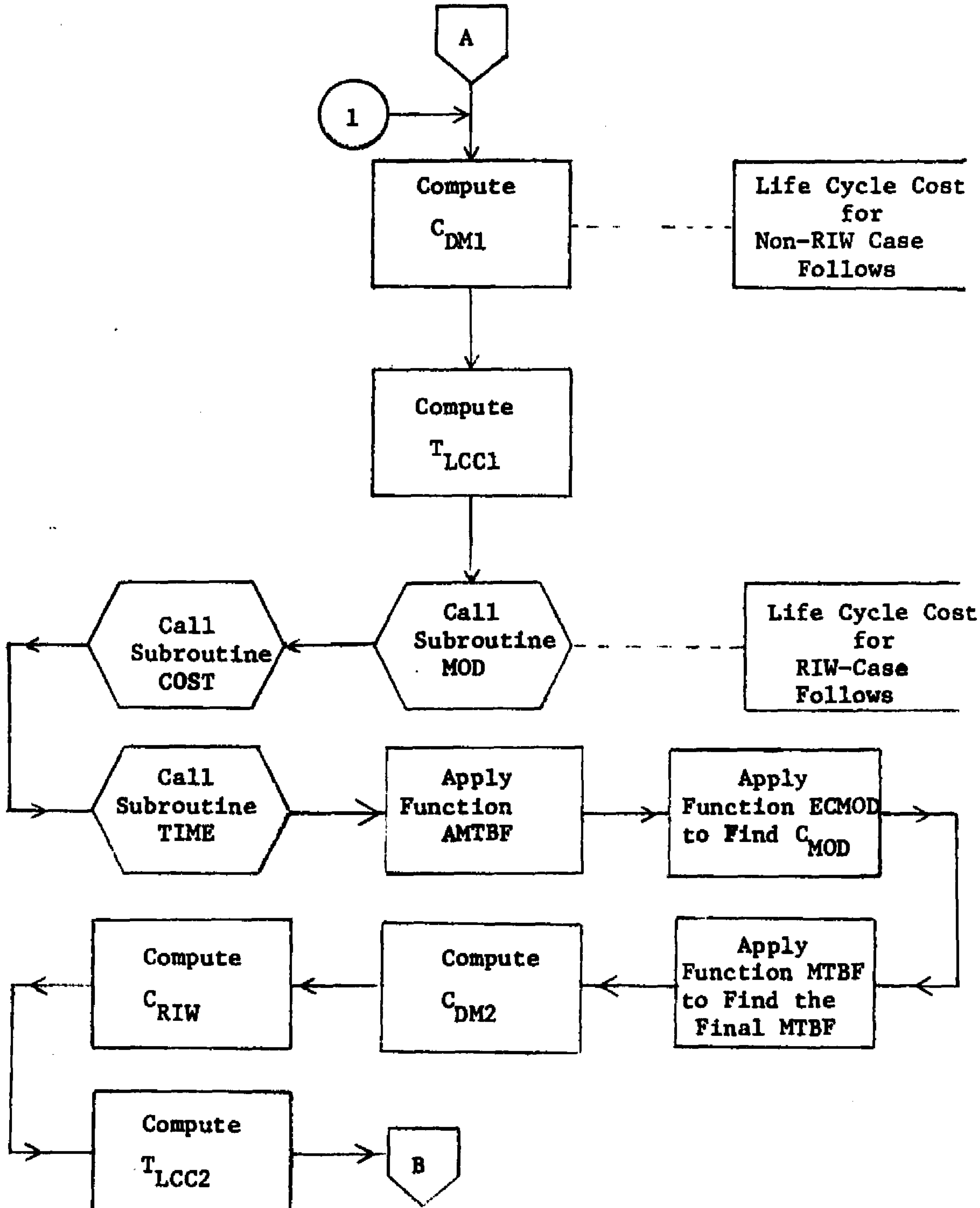

Figure 13 (Continued)

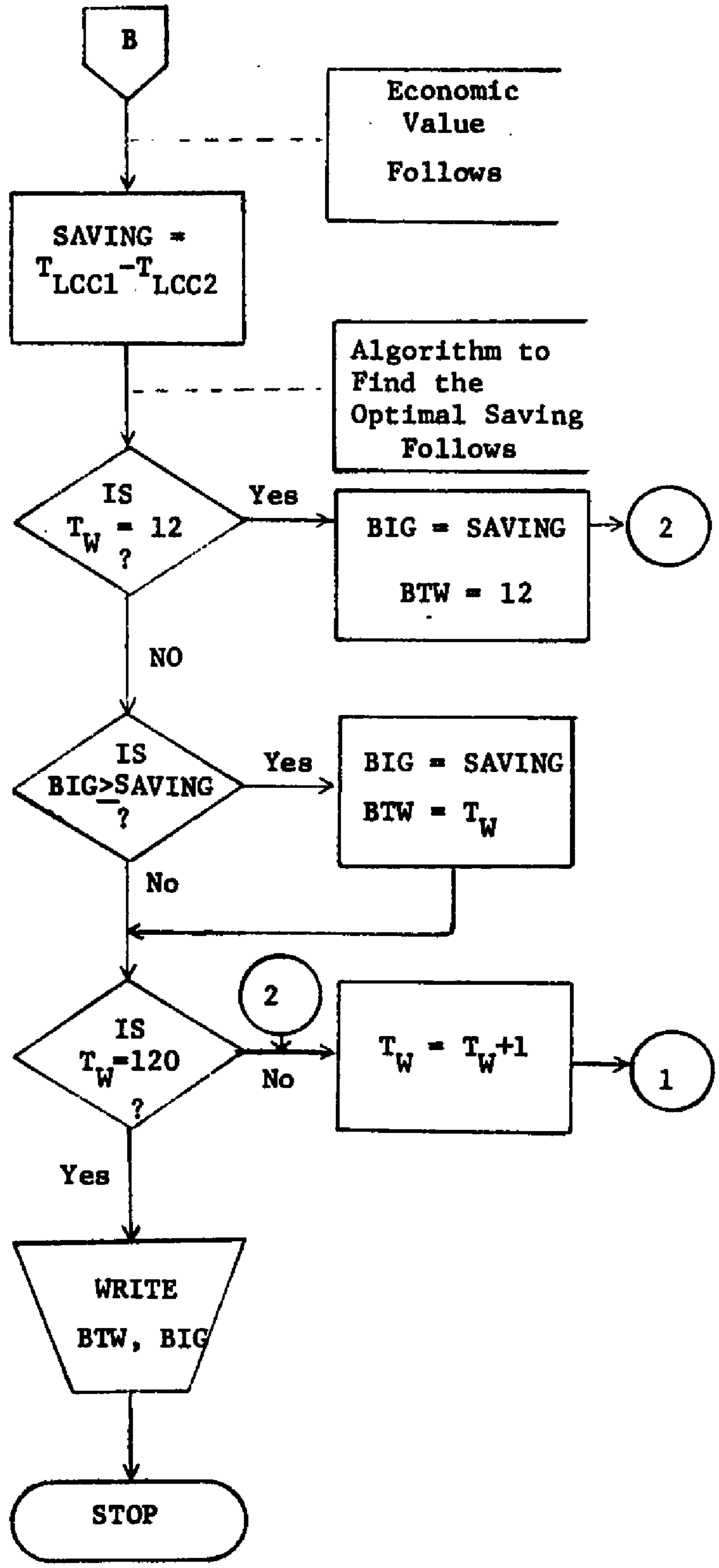

Figure 13 (Continued)

APPENDIX B

DERIVATION OF THE DISCRIMINANT MODEL

Assume that the collection of all procurements is classified into two populations, where population 1 represents the universe of all procurements in which a RIW contract is not applicable and population 2 is the universe of all procurements in which a RIW is applicable. An element belonging to either of the two populations will be denoted in terms of a p-component vector of measurements taken on the item. This vector of measures will be referred to as a procurement vector throughout the discussion which follows. The means of both populations will be respectively denoted as $\vec{\mu}_1$ and $\vec{\mu}_2$. In addition, D_1 and D_2 will be utilized to respectively indicate the covariance matrices of populations 1 and 2.

The objective of this section is to determine a vector $\vec{c}$ to represent the coefficients of the linear discriminant function f of p-variables. This vector of coefficients will be obtained so that the resulting function f will minimize the expected number of misclassifications.

The discussion which follows is divided into two main subdivisions. One subdivision deals with the case where the population parameters are known while the other treats the case where the population parameters are unknown.

KNOWN POPULATION PARAMETERS

In some cases the two populations of interest are well established and their population parameters are known. There are various reasons for justifying such an assumption. For example, in the past many observations may have been taken from the populations and it may be felt that for all intents and purposes that the means and covariance matrices are well established.

Suppose that the two multivariate populations have distributions that are defined by probability density functions g_1 and g_2. Let p_1 and p_2, respectively, indicate the a priori probabilities that an arbitrary vector $\vec{x}$, which represents the p-measures of some unclassified procurement, belongs to either population 1 and 2. Finally, let c_{ij} for $i \neq j$ denote the loss (sometimes called the regret or opportunity loss; $c_{ij} \geq 0$) associated with classifying a procurement represented by $\vec{x}$ into population i when in fact the correct decision should be to classify $\vec{x}$ into population j. For i=j let $c_{ij}=0$.

Classifying into the Two Populations

The decision-theoretic approach to making decisions is often based upon the premise that a classification be made which minimizes the average loss or risk. If a_1 and a_2 are used to denote the actions of classifying a given procurement into populations 1 and 2, respectively, then the risk R can be defined as

$$\begin{aligned} R &= c_{12}\text{Prob}(a_1, \vec{x}\varepsilon \text{ Pop.2}) + c_{21}\text{Prob}(a_2, \vec{x}\varepsilon \text{ Pop.1}) \\ &= c_{12}\text{Prob}(a_1|\vec{x}\varepsilon \text{ Pop.2})p_2 + c_{21}\text{Prob}(a_2|\vec{x}\varepsilon \text{ Pop.1})p_1 \qquad (1) \end{aligned}$$

Next the regions of classification A_1 and A_2 must be respectively defined for populations 1 and 2. These regions have the property that if an arbitrary procurement vector $\vec{x}$ falls into region A_1, then the procurement is classified as belonging to population i for i = 1 or 2. The problem associated with selecting the two regions is that they must be defined so as to minimize the risk R.

Since the density functions of populations 1 and 2 are given as g_1 and g_2, equation (1) can be reexpressed as follows.

$$R = c_{12}\text{Prob}(\vec{x}\varepsilon A_1|\vec{x}\varepsilon\text{Pop.}2)p_2 + c_{21}\text{Prob}(\vec{x}\varepsilon A_2|\vec{x}\varepsilon\text{Pop.}1)p_1$$

$$= c_{12}p_2\int_{A_1} g_2(\vec{x})\, d\vec{x} + c_{21}p_1\int_{A_2} g_1(\vec{x})\, d\vec{x}. \tag{2}$$

Since $\int_{A_1} g_1(\vec{x})\, d\vec{x} + \int_{A_2} g_1(\vec{x})\, d\vec{x} = 1$ equation (2) can be expressed as

$$R = \int_{A_1}[c_{12}p_2g_2\ (\vec{x}) - c_{21}p_1g_1\ (\vec{x})]\ dx + c_{21}p_1 \tag{3}$$

It is now clear that the risk R will be minimized if A_1 is selected to include all vectors $\vec{x}$ for which

$$c_{12}p_2g_2\ (\vec{x}) - c_{21}p_1g_1\ (\vec{x}) \leq 0 \tag{4}$$

while A_2 is defined to include those vectors $\vec{x}$ for which the reverse inequality holds.[1]

Thus the minimum risk rule is to classify an arbitrary procurement vector $\vec{x}$ into population 1 if

$$\frac{g_1(\vec{x})}{g_2(\vec{x})} \geq \left(\frac{c_{12}}{c_{21}}\right)\left(\frac{p_2}{p_1}\right) = \text{constant} \tag{5}$$

Two Normal Populations

Let us now assume that the two populations of procurement types are normally distributed with means $\vec{\mu}_1$, $\vec{\mu}_2$ and covariance

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[1]Press, S. James, Applied Multivariate Analysis, (New York: Holt Rinehart and Winston, Inc., 1972), pp. 371-372.

matrixes D_1 and D_2. Thus the minimum risk rule defined by equation (5) can be revised if g_1 and g_2 are replaced with the given distributions. Thus $\vec{x}$ belongs to population 1 if

$$\frac{|D_1|^{-\frac{1}{2}}\exp[-.5(\vec{x}-\vec{\mu}_1)'\ D_1^{-1}\ (\vec{x}-\vec{\mu}_1)]}{|D_2|^{-\frac{1}{2}}\exp[-.5(\vec{x}-\vec{\mu}_2)'\ D_2^{-1}\ (\vec{x}-\vec{\mu}_2)]} \geq \left(\frac{c_{12}}{c_{21}}\right)\left(\frac{p_2}{p_1}\right) . \tag{6}$$

Applying the natural logarithm to the inequality given by (6) and simplifying the resulting inequality implies that $\vec{x}$ is classified into population 1 if

$$(\vec{x}-\vec{\mu}_2)'\ D_2^{-1}\ (\vec{x}-\vec{\mu}_2) - (\vec{x}-\vec{\mu}_1)'\ D_1^{-1}\ (\vec{x}-\vec{\mu}_1) \geq 2\log\left(\frac{|D_1|^{\frac{1}{2}}p_2c_{12}}{|D_2|^{\frac{1}{2}}p_1c_{21}}\right) \tag{7}$$

Usual Assumption

In most applications of the discriminant model researchers assume that $c_{12} = c_{21}$ (loses due to misclassification are equal), $p_1=p_2$ (prior probabilities are equal) and $D_1=D_2=D$(the populations have equal covariance matrics).[2] Given these assumptions the right-hand side of the inequality in (7) becomes zero and the left-hand side simplifies to give the rule that follows:

Classify the procurement represented by $\vec{x}$ into population 1 if

$$[(\vec{\mu}_1-\vec{\mu}_2)'D^{-1}]\ \vec{x} \geq .5[\vec{\mu}_1'D^{-1}\vec{\mu}_1 - \vec{\mu}_2'D^{-1}\vec{\mu}_2] . \tag{8}$$

Because the parameter values are all known, both expressions in the brackets of inequality (8) can be computed and the inequality tested.

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[2]If the necessary information is available to compute estimates for p_1 and p_2 then the assumption $p_1=p_2$ is not required.

Note that the expression in the bracket of the left-side of inequality (8) yields the vector of coefficients of function f. In addition the expression on the right side of inequality (8) represents the break-point of the classification scheme.

UNKNOWN POPULATION PARAMETERS

In this section the population are assumed to be unknown, which is usually the case. As in the previous section our goal is to find a vector of coefficients to complete the definition of the discriminant function f.

Arbitrary Distributions

Suppose that the two populations of procurement types have functions with unknown parameter values and further suppose the distributional forms of both populations are known though not necessarily normal. Assume that independent observations from both populations exist. To obtain a classification rule we must first find maximum likelihood estimates of the population parameters by utilizing the samples from both populations. Next the estimates for the population parameters are substituted into inequality (5). If the sample sizes from each population are sufficiently large, the results obtained by this technique should be acceptable.[3]

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[3] *Ibid.*, pp. 374-375.

Normal Populations

Assume that the populations are normally distributed with equal covariance matrices and unknown parameter values. Suppose samples of sizes m_1 and m_2 are chosen from populations 1 and 2, respectively. Our problem is as before to classify an arbitrary procurement vector into population 1 or 2 with minimum risk. The best estimates of the two population mean vectors are the sample mean vectors obtained from the two given samples. Let $\bar{x}_1$ and $\bar{x}_2$ denote the sample mean vectors from populations 1 and 2, respectively.

The sample covariance matrix S is used to estimate the population covariance matrix D. The matrix S is defined as

$$(m_1 + m_2 - 2)S = \sum_{i=1}^{m_1} (\vec{x}_{1i} - \bar{x}_1)(\vec{x}_{1i} - \bar{x}_1)' + \sum_{i=1}^{m_2} (\vec{x}_{2i} - \bar{x}_2)(\vec{x}_{2i} - \bar{x}_2)' \qquad (9)$$

where $\vec{x}_{1i}$ and $\vec{x}_{2i}$ represent arbitrary sample vectors from populations 1 and 2, respectively. Substituting these estimates into inequality (8) and simplifying yields

$$\begin{aligned} [(\bar{x}_1 - \bar{x}_2)' S^{-1}]\vec{x} &\geq .5[\bar{x}_1' S^{-1} \bar{x}_1 - \bar{x}_2' S^{-1} \bar{x}_2] \\ &\geq .5[(\bar{x}_1 + \bar{x}_2)' S^{-1} (\bar{x}_1 - \bar{x}_2)] \end{aligned} \qquad (10)$$

The expression in the brackets of the left-side of inequality (10) defines the vector of estimates of the coefficients of the function f while the expression on the right-side of the inequality defines the cut-point for the two populations.[4]

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[4]Anderson, T.W., An Introduction to Multivariate Statistical Analysis (New York: John Wiley, 1958), p. 137.

APPENDIX C

MONTE CARLO GENERATOR OF PROCUREMENTS

```
C-----------------------------------------------------------------
C     ***** PROGRAM GENERATES MEASURES FOR KEY VARIABLES *****
C-----------------------------------------------------------------
      DIMENSION CF1(9),CF2(5),A1(9),A2(5)
C-----------------------------------------------------------------
C     VECTORS A1,CF1 REPRESENT THE  CUMULATIVE DISTRIBUTION FOR UPRICE
C     VECTORS A2,CF2 REPRESENT THE  CUMULATIVE DISTRIBUTION FOR XMTBF
C-----------------------------------------------------------------
      DATA CF1/0.,.75,.8813,.9063,.926,.9435,.9625,.9875,1./
      DATA CF2/0.,.786,.878,.947,1./
      DATA A1/200.,4015.,7830.,11645.,15460.,19275.,23090.,26905.,
     130720./
      DATA A2/51.,1000.,2000.,6000.,16400./
C-----------------------------------------------------------------
C     ***** 200 SETS OF MEASURES FOR THE KEY VARIABLES ARE GENERATED
C-----------------------------------------------------------------
      DO 1 I=1,200
      KUNITS=RANVAR(2,380,DUMMY)+.5
      IF(KUNITS.LT.25)KUNITS=25
      IF(KUNITS.GT.9999)KUNITS=9999
      OUNITS=KUNITS
      OHOURS=RANVAR(3,56.,16.4)
      X=1.+RANVAR(1,.02,.50)
      TP=RANVAR(1,.5,3.25)
      UPRICE=GEN(CF1,A1,9)
      XMTBF=GEN(CF2,A2,5)
      SMTBF=X*XMTBF
      Y=RANVAR(1,.05,1.05)
```

```
      CF1=Y*UPRICE
      CF2=9.452+.098*CF1
      Q=RANVAR(1,.60,.90)
      CF=Q*CF1
C     *****  KEY VALUES ARE WRITTEN ONTO A MAG TAPE  *****
       WRITE(7,2)OUNITS,OHOURS, XMTBF,SMTBF,UPRICE,CF1,CF2,CF,TP
    2 FORMAT(F6.0,F5.2,F7.1,F7.1,F8.2,F8.2,F8.2,F8.2,F4.2)
    1 CONTINUE
      REWIND 7
      ENDFILE 7
      STOP
      END
```

```
C-----------------------------------------------------------------
      FUNCTION GEN(CF,A,K)
C-----------------------------------------------------------------
C     SUBPROGRAM GENERATES RANDOM VARIATES FROM CUM. DISTRIBUTIONS
C     ALL VALUES ARE INTERPOLATED LINEARLY
C-----------------------------------------------------------------
      DIMENSION CF(9),A(9)
      XNUMB=RANVAR(1,0.,1.)
      DO 1 I=1,K
      IF(XNUMB.LT.CF(I))GO TO 2
    1 CONTINUE
    2 F=(XNUMB-CF(I-1))*(CF(I)-CF(I-1))
      GEN=F*(A(I)-A(I-1))+A(I-1)
      RETURN
      END


      FUNCTION RBBRNG(N)
C-----------------------------------------------------------------
C     THIS SUBROGRAM SEEDS GENERATOR AND FINDS RANDOM NO. BETWEEN 0,1
C-----------------------------------------------------------------
      IF(N.NE.0)NSEED=987547
      NSEED=NSEED*262147
      RBBRNG=DABS(NSEED/3435973836.7D+1)
      RETURN
      END
```

```
C-----------------------------------------------------------------
      FUNCTION RANVAR(KIND,A,B)
C-----------------------------------------------------------------
C     THE FOLLOWING IS A LIST OF PARAMETER DEFINITIONS
C
C-----------------------------------------------------------------
C       KIND   NAME OF DIST       ARGUMENT A       ARGUMENT B
C         1     UNIFORM            LOW LIMIT        UPPER LIMIT
C         2     EXPONENTIAL        MEAN             DUMMY
C         3     NORMAL             MEAN             STANDARD DEV.
C-----------------------------------------------------------------
      IF(ISW.EQ.1)GO TO 10
      RAN=RBBRNG(0)
      ISW=1
   10 GO TO(107,207,307),KIND
C-----------------------------------------------------------------
C     *****  UNIFORM DIST   *****
C-----------------------------------------------------------------
  107 RAN=RBBNG(0)
      RANVAR=RAN*(B-A)+A
      RETURN
C-----------------------------------------------------------------
C     *****  EXPONENTIAL DIST  *****
C-----------------------------------------------------------------
  207 EX=A
      RAN=RBBRNG(0)
      RANVAR=-ALOG(1.0-RAN)*EX
```

```
      RETURN
C-----------------------------------------------------------------
C     *****  NORMAL DIST  *****
C-----------------------------------------------------------------
  308 R1=RBBRNG(0)
      R2=RBBRNG(0)
      R=(-2.0*ALOG(R1))**.5*COS(2.0*3.1416*R2)
      RANVAR=R*B+A
      RETURN
      END
```

APPENDIX D

NON-RIW AND RIW PROCUREMENT GROUPS

Table 13

Population 1 - Procurements Without RIW Contracts

U_0	H_0	θ_i	θ^*	T_P	P	C_{F1}	C_{F2}	C_F
599	52.3	199.8	244.9	2.82	294.40	163.62	25.49	138.95
25	45.5	1,001.5	1,001.5[a]	0.63	1,738.00	125.57	21.76	99.58
895	47.3	6,006.6	6,006.6	2.95	4,060.90	2,932.24	296.81	2,342.03
433	48.6	452.5	523.5	0.59	1,716.30	90.46	18.32	76.77
2,243	65.2	572.5	636.2	1.15	4,043.10	4,004.02	401.85	3,183.19
118	38.4	511.9	650.9	0.77	1,875.50	913.18	98.94	805.20
159	49.3	585.9	816.2	1.52	501.30	400.66	48.72	354.51
308	61.3	320.2	324.0	3.08	763.20	86.19	17.90	61.39
635	51.9	2,013.4	2,655.4	2.95	720.40	614.52	69.68	523.81
122	92.7	398.5	509.5	0.93	4,051.50	1,018.88	109.30	911.96
364	44.3	260.8	302.1	1.31	371.90	103.41	19.59	88.03

[a]In those cases where the economic value model found any improvement in reliability not feasible, the specified MTBF is assigned the initial MTBF.

Table 13 (Continued)

U_0	H_0	θ_i	θ*	T_P	P	C_{F1}	C_{F2}	C_F
836	70.6	420.5	445.8	2.20	1,174.30	166.91	25.81	149.6
543	70.0	1,000.2	1,037.7	2.86	1,400.40	1,406.36	147.28	1,109.92
320	50.8	182.9	248.5	0.56	1,141.40	171.34	26.24	153.75
236	59.3	324.1	334.1	1.79	2,046.20	110.00	20.23	90.02
256	63.7	513.2	581.1	2.26	2,295.10	2,117.10	216.93	1,785.40
1,611	46.4	461.0	544.0	1.76	1,906.40	1,363.88	143.11	1,208.80
407	53.9	151.1	186.0	2.03	614.30	97.05	18.96	74.52
416	38.9	2,015.7	2,146.4	3.22	1,238.10	1,143.91	121.55	971.39
30	85.2	81.2	98.8	0.85	633.80	588.25	67.10	512.68
638	34.2	6,023.2	6.223.1	3.04	535.60	556.71	64.01	495.03
128	25.3	256.9	315.9	0.64	4,079.70	1,962.97	201.82	1,725.05
435	31.4	1,002.3	1,089.8	1.87	19,275.10	13,049.16	1,288.27	11,149.20
38	65.2	195.0	215.2	1.77	1,132.50	160.02	25.13	134.92
25	52.3	2,005.2	2,005.2	2.55	1,062.40	151.68	24.32	113.14

Table 13 (Continued)

U_0	H_0	θ_i	θ^*	T_P	P	C_{F1}	C_{F2}	C_F
33	89.5	2,014.6	2,706.4	3.21	2,013.00	338.04	42.58	295.64
77	41.1	268.0	325.7	0.83	775.10	87.76	18.05	78.93
103	48.1	122.7	147.9	1.34	4,071.00	1,555.24	161.87	1,340.23
297	54.5	6,017.0	6,017.0	2.73	350.90	149.66	24.12	116.68
737	48.5	321.8	390.4	0.99	796.90	84.18	17.70	74.08
40	40.1	459.9	489.9	0.76	342.70	107.01	19.94	80.47
432	75.7	569.0	569.0	2.51	251.00	14.12	10.84	9.37
420	19.7	2,000.3	2,000.3	1.75	580.90	73.82	16.69	56.47
935	76.0	6,025.2	6,115.1	2.04	1,125.70	1,131.91	120.38	905.80
855	21.7	503.7	503.7	0.75	2,215.00	156.36	24.78	113.27
425	63.0	116.1	118.1	1.21	1,904.60	312.64	40.09	237.20
910	38.3	1,002.1	1,017.6	0.81	1,243.70	232.02	32.19	171.32
215	75.5	397.0	442.1	1.87	1,149.20	290.10	32.88	227.20
274	67.6	487.5	499.4	0.65	237.50	155.88	24.73	135.89

Table 13 (Continued)

U_0	H_0	θ_i	θ^*	T_P	P	C_{F1}	C_{F2}	C_F
95	49.1	6,017.4	7,167.1	2.59	1,091.30	791.12	86.98	654.43
366	58.8	431.2	570.5	1.52	2,173.80	1,701.60	176.21	1,523.47
878	59.0	6,005.2	6,005.2	1.63	847.10	234.00	32.38	142.87
108	57.8	422.8	450.4	0.56	1,300.80	678.38	75.93	546.33
164	16.8	2,001.2	2,244.9	2.31	699.10	639.25	72.10	528.41
142	51.7	113.1	119.5	0.87	1,497.20	942.39	101.81	771.20
716	63.1	283.4	333.4	3.03	1,819.10	1,842.29	190.00	1,642.44
790	67.0	488.0	498.0	1.22	699.30	674.41	75.54	558.26
149	58.3	6,028.7	6,028.7	0.76	2,034.70	579.54	66.25	417.27
61	42.7	71.6	84.4	0.85	4,065.90	897.57	97.41	734.42
25	55.1	164.5	215.7	0.90	1,410.40	1,085.93	115.87	961.28
331	47.1	311.8	365.5	0.66	1,235.50	858.88	93.62	768.46
25	53.6	1,002.0	1,302.7	1.09	797.00	619.20	70.13	540.96
736	71.3	98.8	106.0	1.52	487.10	371.42	45.85	317.80

Table 13 (Continued)

U_0	H_0	θ_i	θ^*	T_P	P	C_{F1}	C_{F2}	C_F
160	51.2	273.3	281.5	3.09	4,074.20	1,823.06	188.11	1,603.90
195	98.9	498.4	591.7	3.09	251.90	154.73	24.62	129.24
70	57.3	123.8	133.7	2.23	1,274.00	109.44	20.18	95.40
56	67.5	6,007.2	6,007.2	1.07	598.10	334.08	42.19	241.57
46	53.3	368.1	391.1	1.84	1,131.00	645.26	722.69	570.93

Table 14

Population 2 - Procurements With RIW Contracts

U_0	H_0	θ_1	θ^*	T_P	P	C_{F1}	C_{F2}	C_F
215	59.2	1,005.0	1,421.1	1.57	997.80	496.16	58.08	334.67
290	66.9	6,023.7	6,237.4	0.70	2,269.90	1,409.18	147.55	987.14
783	27.9	244.2	350.2	0.82	1,528.90	482.41	56.73	300.36
701	44.7	223.2	295.7	1.06	15,460.60	3,938.38	395.41	3,182.87
25	70.1	164.1	185.9	2.45	1,265.30	392.82	47.95	259.07
711	64.8	267.7	375.9	0.66	4,073.00	2,095.65	214.83	1,633.15
100	45.5	1,005.3	1,169.6	2.16	1,826.60	190.17	28.09	118.78
1,460	64.8	580.0	645.8	2.42	667.40	557.53	64.09	389.54
29	59.3	1,008.3	1,503.4	0.71	2,189.90	2,289.70	233.84	1,436.60
201	68.9	6,025.6	7,735.2	1.49	1,266.80	829.21	90.71	668.42
171	48.8	556.0	765.1	2.61	401.40	299.25	38.78	186.74
94	44.9	2,015.6	2,205.5	1.84	898.90	220.63	31.07	160.06
215	51.8	246.5	364.8	0.91	958.40	827.01	90.05	685.66

Table 14 (Continued)

U_0	H_0	θ_i	θ^*	T_P	P	C_{F1}	C_{F2}	C_F
41	75.0	415.2	550.1	1.13	11,645.20	10,933.56	1,080.94	7,453.01
173	43.7	540.5	591.4	2.94	7,831.00	653.60	73.50	413.18
1,170	45.7	184.8	215.6	1.74	1,215.40	203.67	29.41	413.18
751	81.7	171.1	232.5	0.73	642.00	304.56	39.30	183.17
62	52.7	105.8	127.6	1.42	1,667.10	373.15	46.02	256.97
412	64.5	192.8	257.4	1.88	4,080.40	4,014.44	402.87	3,235.39
218	60.3	154.6	172.5	2.30	2,125.30	1,758.35	181.77	1,363.97
271	72.4	636.8	894.7	2.07	2,074.90	1,793.66	185.23	1,173.78
205	47.6	275.6	402.4	3.13	1,670.10	802.72	88.12	511.98
378	57.7	6,015.4	6,748.1	2.98	4,044.70	2,492.22	253.69	1,960.24
383	30.6	6,010.6	7,511.4	3.06	512.80	426.19	51.22	256.90
937	61.5	485.5	708.8	1.52	235.00	104.61	19.70	74.91
49	68.5	439.8	624.5	2.18	661.00	414.73	50.10	345.08
127	63.3	752.1	630.5	1.57	4,040.20	2,647.88	268.94	1,652.94

Table 14 (Continued)

U_0	H_0	θ_i	θ*	T_P	P	C_{F1}	C_{F2}	C_F
26	60.4	485.3	689.6	1.75	920.30	267.59	35.68	215.53
25	37.4	279.9	370.6	3.20	738.60	145.63	23.72	92.68
345	48.5	569.4	588.1	1.55	1,088.60	1,105.22	117.76	816.87
287	45.5	585.4	596.4	1.48	435.60	440.07	52.58	291.75
1,353	81.5	272.9	349.6	1.63	1,442.70	250.99	34.05	194.16
336	37.0	408.6	578.6	2.02	2,344.40	1,003.28	107.77	777.41
38	67.2	357.6	486.3	1.18	7,831.10	4,356.54	436.39	3,479.10
285	44.8	1,006.4	1,095.8	2.30	662.80	246.24	33.58	175.04
619	58.9	392.9	413.3	2.66	1,374.00	677.13	75.81	514.33
599	67.7	295.2	440.1	1.85	561.80	492.40	57.71	297.14
379	33.5	368.1	480.4	2.08	3,036.50	1,468.77	153.39	1,009.37
714	35.6	181.2	245.5	0.76	4,058.40	2,162.57	221.38	1,601.24
154	49.9	160.8	231.6	1.86	7,831.80	8,136.73	806.85	6,368.14
530	51.9	1,004.8	1,348.4	3.24	258.50	216.11	30.63	146.57

Table 14 (Continued)

U_0	H_0	θ_i	θ^*	T_P	P	C_{F1}	C_{F2}	C_F
127	57.6	1,008.1	1,380.1	2.71	2,063.90	1,950.24	200.58	1,619.00
151	53.0	391.5	458.6	2.27	512.20	149.97	24.15	109.53
596	64.4	104.0	106.9	1.63	15,460.70	14,041.66	1,385.54	10,572.22
811	49.6	1,001.9	1,470.8	1.76	4,061.40	1,934.68	199.05	1,570.85
243	54.1	395.7	592.8	2.57	1,524.50	1,507.80	157.22	1,142.17
193	62.4	222.3	291.2	1.43	1,716.80	1,127.40	119.94	820.68
25	29.8	633.9	774.0	2.80	4,036.10	933.10	100.90	593.63
902	74.6	2,005.8	2,758.0	2.02	258.10	122.10	21.42	96.35
25	56.7	397.2	406.1	2.11	776.20	793.40	87.21	557.93
444	39.3	241.2	270.0	1.35	904.50	665.46	74.67	481.08
298	70.9	2,012.1	2,575.5	1.91	347.70	341.67	42.94	215.87
1,323	65.6	302.6	375.8	1.39	469.30	432.43	51.83	284.37
445	89.1	1,006.5	1,398.0	1.28	1,966.10	1,467.72	153.29	1,086.37
291	56.4	1,004.2	1,098.8	1.97	26,905.40	2,690.58	273.13	1,786.69

Table 14 (Continued)

U_0	H_0	θ_i	θ^*	T_P	P	C_{F1}	C_{F2}	C_F
119	61.1	85.2	124.7	1.72	1,122.80	979.61	105.45	779.56
521	51.4	537.0	588.6	2.66	2,245.60	884.16	96.10	583.13
25	53.3	230.4	293.5	2.66	1,707.50	1,287.17	135.20	786.85
305	49.4	76.0	104.1	0.97	1,389.30	708.82	78.92	523.05
364	65.1	403.9	580.8	2.79	4,054.20	2,701.12	274.16	1,657.16
331	39.4	537.9	722.9	1.63	1,321.50	579.71	66.26	427.38
531	71.1	391.6	495.4	1.32	1,256.50	142.07	23.37	96.69
1,183	44.3	53.5	67.2	0.98	1,893.80	1,329.40	139.73	820.83
92	52.8	248.1	342.1	2.81	1,073.10	953.92	102.94	759.98
183	77.1	236.2	249.9	0.61	2,051.00	785.29	86.41	525.69
189	77.4	227.8	241.9	0.56	2,310.60	1,926.13	198.21	1,483.81
166	58.6	563.6	632.2	1.32	2,105.80	353.05	44.05	267.06
27	52.0	312.4	442.4	2.11	15,460.80	6,111.78	608.41	5,085.90
108	35.7	235.8	349.5	2.92	354.30	371.62	45.87	247.87

Table 14 (Continued)

U_0	H_0	θ_i	θ^*	T_P	P	C_{F1}	C_{F2}	C_F
163	32.0	540.7	623.1	1.29	4,075.90	589.90	67.26	470.62
862	61.6	433.4	637.5	0.57	1,976.50	1,410.70	147.70	1,113.57
96	72.3	458.8	489.5	1.51	2,183.00	1,619.96	123.32	762.04
858	82.2	288.0	403.5	2.28	23,091.20	16,878.52	1,663.55	14,045.40
560	79.3	1,001.0	1,366.4	1.32	898.70	570.31	65.34	399.31
514	73.9	283.2	423.1	0.54	23,090.60	14,801.09	1,459.96	12,183.10
352	47.2	496.5	547.1	2.15	204.90	92.48	18.52	61.51
343	72.3	226.5	282.5	0.96	2,225.20	596.71	67.93	460.66
344	67.8	2,002.6	2,274.7	2.80	1,917.20	1,488.26	155.30	1,058.35
143	44.8	581.9	600.4	3.03	1,620.00	399.18	48.57	289.36
117	43.1	538.4	680.0	2.75	1,773.90	1,808.68	186.68	1,119.43
85	70.8	1,002.0	1,397.8	2.07	1,304.80	486.76	57.15	389.38
25	86.7	1,001.0	1,252.3	0.99	1,918.40	448.36	53.39	277.83
25	50.4	1,008.3	1,469.1	2.95	4,065.60	3,923.03	393.31	2,817.98

Table 14 (Continued)

U_0	H_0	θ_i	θ^*	T_P	P	C_{F1}	C_{F2}	C_F
348	91.0	440.0	650.3	0.66	1,244.10	255.70	34.51	182.08
168	68.1	200.1	251.1	1.49	1,842.70	151.28	24.28	108.64
29	55.9	200.8	232.9	2.79	1,295.50	908.21	98.46	571.77
499	38.5	117.5	172.7	1.87	1,839.80	657.55	73.89	521.99
837	64.1	619.9	900.7	1.53	1,215.90	813.72	89.20	629.75
37	76.0	152.9	219.3	0.54	1,153.00	267.85	35.70	186.73
136	52.3	317.4	331.0	1.63	509.20	447.87	53.34	294.49
143	69.8	519.0	771.8	0.82	350.50	304.14	39.26	218.02
1,464	37.9	560.6	826.3	1.09	4,025.10	3,551.10	357.46	3,044.59
385	82.3	145.8	175.4	1.36	1,484.80	1,273.82	134.29	879.05
1,023	36.5	103.3	122.1	1.95	516.40	327.64	41.56	210.15
25	31.2	2,005.0	2,650.6	0.82	1,224.20	794.59	87.32	589.19
443	56.0	1,005.5	1,232.7	1.35	233.10	237.71	32.75	157.48
279	61.1	620.1	737.3	1.95	1,138.10	593.47	67.61	410.30

Table 14 (Continued)

U_0	H_0	θ_i	θ^*	T_P	P	C_{F1}	C_{F2}	C_F
27	32.4	416.7	508.0	1.74	1,408.40	1,280.62	134.95	934.23
148	21.1	140.7	145.1	2.12	595.50	338.29	42.60	222.96
271	64.5	387.3	534.5	2.63	474.30	228.65	31.86	181.77
25	64.6	104.4	143.7	2.52	11,645.20	3,459.36	348.47	2,945.19
618	43.2	2,003.0	2,812.2	2.97	771.90	708.22	78.86	593.66
323	33.5	296.0	428.0	0.73	1,662.10	1,127.41	119.94	921.99
459	53.0	164.0	220.7	1.47	829.60	811.80	89.01	537.79
846	66.3	146.0	164.6	3.01	993.00	786.13	86.49	566.95
682	83.3	405.5	455.4	1.43	467.90	166.50	25.77	105.52
30	64.0	129.5	147.4	3.01	1,097.70	265.98	35.52	167.65
128	78.4	1,000.9	1,145.0	2.62	1,021.10	244.27	33.39	148.21
67	47.4	133.4	197.7	1.94	767.70	382.94	46.98	264.43
39	64.3	152.0	198.4	3.15	4,071.80	2,574.36	261.74	1,756.91
199	69.8	456.7	600.6	1.86	1,622.20	1,362.52	142.98	881.02

Table 14 (Continued)

U_0	H_0	θ_i	θ^*	T_P	P	C_{F1}	C_{F2}	C_F
567	54.8	2,004.9	2,758.7	0.59	1,538.60	716.85	79.70	549.62
199	73.3	320.7	451.5	1.24	1,181.10	938.29	101.40	641.49
177	82.3	246.9	319.0	1.00	248.60	79.94	17.29	48.15
255	58.8	459.5	688.3	2.71	828.50	614.14	69.64	538.70
102	55.5	221.4	229.0	1.99	4,065.30	1,352.44	141.99	912.05
25	15.4	371.3	525.4	0.91	1,671.30	169.95	26.11	110.87
134	67.2	1,004.7	1,500.0	0.54	4,032.00	323.69	41.17	221.51
25	75.3	273.5	277.9	1.76	2,025.70	1,048.43	112.20	789.50
499	55.1	411.7	605.2	0.52	1,359.10	1,080.07	115.30	913.77
93	27.9	156.1	181.9	2.17	1,778.30	427.38	51.33	308.86
518	67.5	2,009.8	2,902.2	2.05	2,108.50	1,190.95	126.17	960.34
543	37.6	199.2	297.2	1.82	255.20	174.63	26.54	126.08
60	67.6	249.5	357.8	2.04	4,075.50	1,183.27	125.41	767.99
457	69.6	381.9	483.9	2.80	1,164.00	414.85	50.11	270.01

Table 14 (Continued)

U_0	H_0	θ_i	θ^*	T_P	P	C_{F1}	C_{F2}	C_F
96	57.0	1,000.8	1,259.0	0.66	525.00	135.69	22.75	97.89
305	79.0	573.4	778.7	1.24	332.90	88.65	18.14	63.82
666	50.7	502.0	515.2	1.91	2,111.00	868.19	94.53	622.77
177	43.3	2,002.5	2,923.6	1.66	4,030.90	1,356.39	142.38	921.66
105	33.9	544.1	642.0	2.61	1,366.30	567.70	65.09	364.82
218	21.2	1,004.7	1,154.4	2.92	4,062.50	2,734.94	277.48	1,811.91
168	96.8	389.3	510.8	0.83	2,328.60	1,474.46	153.95	1,070.04
102	66.6	150.9	195.7	2.44	23,090.40	3,558.22	358.16	2,755.14
208	69.5	222.4	279.3	2.00	4,069.50	1,627.96	168.99	1,059.22
1,648	74.9	269.1	349.6	3.24	4,034.60	3,657.90	367.93	2,752.72
408	56.6	106.4	144.8	3.11	416.70	384.47	47.13	241.66
210	41.7	236.1	332.9	1.15	2,289.50	2,211.76	226.20	1,815.89
415	64.2	157.0	216.8	3.07	1,284.50	403.05	48.95	278.60
302	54.8	181.7	252.6	0.72	1,847.20	332.06	41.99	234.66

Table 14 (Continued)

U_0	H_0	θ_i	θ^*	T_P	P	C_{F1}	C_{F2}	C_F
1,281	67.6	191.9	201.2	1.96	666.50	255.91	34.53	176.28
580	82.4	568.6	739.2	2.72	477.90	483.74	56.86	292.87
303	65.6	164.3	180.2	1.88	1,465.50	1,436.81	150.26	955.21

APPENDIX E

Ranked Discriminant Function Values

Table 15

Ranked Discriminant Functional Values

Rank	First Group Values	Second Group Values	First Group Item No.	Second Group Item No.
1	.00720		42	
2	.00580		3	
3	.00548		48	
4	.00530		29	
5	.00455		34	
6	.00436		57	
7		.00325		2
8	.00280		21	
9	.00253		33	
10	.00211		35	
11	.00164		37	
12	.00150		25	
13	.00117		32	
14	.00074		2	
15	.00008			23
16	.00000		8	
17		-.00074		128
18	-.00088		15	
19		-.00110		79
20	-.00145		12	
21	-.00202		54	

Table 15 (Continued)

Rank	First Group Values	Second Group Values	First Group Item No.	Second Group Item No.
22		-.00205		116
23		-.00209		119
24		-.00220		36
25	-.00228		39	
26		-.00231		12
27	-.00242		36	
28	-.00242		13	
29	-.00256		47	
30		-.00257		55
31	-.00258		19	
32		-.00267		99
33		-.00287		31
34		-.00310		35
35		-.00315		15
36	-.00316		31	
37		-.00332		30
38		-.00335		140
39		-.00342		57
40	-.00349		56	
41		-.00361		65
42	-.00373		40	
43	-.00374		23	

Table 15 (Continued)

Rank	First Group Values	Second Group Values	First Group Item No.	Second Group Item No.
44	-.00374		58	
45		-.00375		50
46	-.00376		43	
47		-.00384		72
48		-.00390		8
49		-.00393		90
50	-.00427		24	
51		-.00438		76
52		-.00452		67
53	-.00453		45	
54	-.00468		38	
55		-.00501		78
56		-.00510		108
57		-.00515		106
58	-.00524		44	
59	-.00526		4	
60	-.00531		5	
61		-.00535		66
62		-.00552		27
63		-.00558		70
64		-.00562		7
65		-.00563		24

Table 15 (Continued)

Rank	First Group Values	Second Group Values	First Group Item No.	Second Group Item No.
66		-.00596		5
67		-.00605		107
68		-.00645		16
69	-.00645		17	
70		-.00653		51
71		-.00659		131
72	-.00668		11	
73		-.00693		10
74		-.00708		48
75		-.00712		121
76		-.00730		142
77	-.00737		16	
78		-.00745		20
79		-.00771		105
80		-.00773		130
81	-.00786		49	
82	-.00787		30	
83		-.00813		97
84		-.00819		86
85	-.00838		51	
86	-.00852		27	
87	-.00857		55	
88	-.00859		53	

Table 15 (Continued)

Rank	First Group Values	Second Group Values	First Group Item No.	Second Group Item No.
89		-.00888		48
90		-.00893		18
91	-.00930		46	
92	-.00930		28	
93	-.00962		18	
94		-.00963		94
95		-.00973		82
96		-.00987		126
97		-.00995		62
98	-.01000		1	
99		-.01015		85
100		-.01021		96
101	-.01030		22	
102		-.01066		125
103		-.01092		77
104		-.01116		98
105		-.01140		134
106		-.01158		52
107	-.01159		10	
108	-.01160		20	
109	-.01165		6	
110		-.01171		32
111		-.01173		53

Table 15 (Continued)

Rank	First Group Values	Second Group Values	First Group Item No.	Second Group Item No.
112	-.01176		9	
113		-.01204		93
114		-.01210		38
115	-.01219		26	
116		-.01224		49
117		-.01240		133
118		-.01260		114
119		-.01284		95
120		-.01298		80
121		-.01302		44
122		-.01307		112
123		-.01307		58
124		-.01323		29
125	-.01337		52	
126		-.01360		141
127		-.01376		4
128		-.01378		61
129		-.01395		110
130		-.01398		91
131		-.01398		127
132		-.01401		41
133		-.01409		132
134		-.01414		47

Table 15 (Continued)

Rank	First Group Values	Second Group Values	First Group Item No.	Second Group Item No.
135		-.01435		111
136		-.01454		74
137	-.01457		41	
138	-.01466		14	
139	-.01471		50	
140		-.01477		135
141		-.01530		102
142		-.01562		14
143		-.01567		81
144		-.01576		39
145		-.01577		101
146		-.01577		100
147		-.01579		34
148		-.01593		138
149		-.01606		139
150		-.01608		54
151		-.01611		122
152		-.01638		11
153		-.01641		117
154		-.01646		42
155		-.01664		1
156		-.01684		33
157		-.01686		3

Table 15 (Continued)

Rank	First Group Values	Second Group Values	First Group Item No.	Second Group Item No.
158		-.01687		129
159		-.01692		104
160		-.01697		19
161		-.01716		28
162	-.01718		7	
163		-.01722		136
164		-.01737		45
165		-.01752		68
166		-.01758		64
167		-.01769		59
168		-.01784		17
169		-.01784		21
170		-.01785		124
171		-.01788		25
172		-.01802		26
173		-.01804		89
174		-.01808		6
175		-.01809		88
176		-.01822		113
177		-.01866		92
178		-.01867		118
179		-.01875		60
180		-.01887		137

Table 15 (Continued)

Rank	First Group Values	Second Group Values	First Group Item No.	Second Group Item No.
181		-.01910		84
182		-.01920		22
183		-.01927		103
184		-.01986		87
185		-.02000		71
186		-.02017		83
187		-.02040		120
188		-.02060		109
189		-.02101		123
190		-.02121		40
191		-.02128		115
192		-.02135		13
193		-.02149		37
194		-.02149		56
195		-.02163		63
196		-.02178		9
197		-.02187		69
198		-.02218		46
199		-.02340		73
200		-.02435		75

VITA

Raymond Otis Folse, son of the late Mr. Emile L. Folse, Sr., and Florence K. Folse, was born on November 17, 1936 in Thibodaux, Louisiana. A graduate of Thibodaux High School in Thibodaux, Louisiana, he began his college career in the fall of 1955. He earned in 1959 a Bachelor of Science degree in science and mathematics education from Nicholls State University in Thibodaux, Louisiana. In 1962 he received a National Science Foundation stipend to support his studies toward a masters degree in mathematics at Louisiana State University in Baton Rouge, Louisiana. During the summer of 1963 Mr. Folse received his Master of Arts degree in mathematics from Louisiana State University. In the fall of 1971 Mr. Folse began his doctoral program at Louisiana State University on a part time basis. During the course of his studies he majored in quantitative methods and minored in mathematics and management (business communication).

His teaching career includes sixteen years of experience. Mr. Folse has been employed by Nicholls State University since the fall of 1964. Initially his duties at Nicholls State University included teaching mathematics. In 1970 he was asked to teach computer science courses as a member of the newly formed computer science department. His training in computer science includes his enrollment in a National Science Foundation institute in computer science for college and university teachers during the summer of 1965. In addition Mr. Folse has attended numerous workshops and seminars sponsored by the major computer equipment manufacturers.